ON DIMENSIONING OF AN OPERATIONALLY HEAT PUMP – NEW APPROACHES TOWARDS EFFICIENT PERFORMANCE

ZUR DIMENSIONIERUNG EINER BETRIEBSOPTIMALEN WÄRMEPUMPE – NEUE WEGE ZUR LEISTUNGSEFFIZIENZ

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ABSTRACT

Presented is a new approach on how to design heat engines optimally. In this context, the problem of an operationally optimal dimensioning of heat pumps is considered as a modelling problem. The design calculus developed herein aims, by means of a criterion-driven assessment of compressor efficiency, to enable both the dimensioning and the validation of system and control paraeters of a corresponding plant model. For this purpose, an associative quantity is drived which, in form of a Riemannian length functional $\boldsymbol{L} \coloneqq \int \sqrt{\boldsymbol{g}_{\boldsymbol{\gamma}(t)}(\dot{\boldsymbol{\gamma}}, \dot{\boldsymbol{\gamma}})}\boldsymbol{dt}$ or, respectively, a Riemannian energy functional $\boldsymbol{E} \coloneqq \int \boldsymbol{g}_{\boldsymbol{\gamma}(t)}(\dot{\boldsymbol{\gamma}}, \dot{\boldsymbol{\gamma}})\, \boldsymbol{dt}$ is endowed with a metric and thus evaluable corresponence. With regard to the current chalenges in the fields of Civil Engineering, Energy and Supply Engineering in general, and with regard to the dimensioning problem discussed here in particular, the proposed new calculation method represents a valuable addition to the established methods. It expands the existing methodological spectrum by adding an additional perspective for dealing with dimensioning tasks, especially where the multi-criteria optimisation problems arise and conventional approaches run up against their limits.

ZUSAMMENFASSUNG

In diesem Beitrag wird ein neues Kalkül zur optimalen Auslegung von Kraftwärmemaschinen erörtert. Speziell in diesem Kontext wird das Problem einer betriebsoptimalen Dimensionierung von Wärmepumpen zunächjst als ein Modelierungsproblem betrachtet. Das hier ausgearbeitete Auslegungskalkül sieht vor, auf Basis einer kriteriell-basierten Effizienzbewertung der Kompressorarbeit, sowohl die Dimensionierung wie auch die Validierung von System- und Regelparametern eines entsprechenden Anlagenmodells vornehmen zu können. Dabei wird zur

Bewertung der Effizienz eine Assoziativgröße abgeleitet, die dann in Gestalt eines Riemann'schen Längenfunktionals $\boldsymbol{L} \coloneqq \int \sqrt{\boldsymbol{g}_{\boldsymbol{\gamma}(t)}(\dot{\boldsymbol{\gamma}}, \dot{\boldsymbol{\gamma}})}\boldsymbol{dt}$ resp. Energiefunktionals $\boldsymbol{E} \coloneqq \int \boldsymbol{g}_{\boldsymbol{\gamma}(t)}(\dot{\boldsymbol{\gamma}}, \dot{\boldsymbol{\gamma}})\, \boldsymbol{dt}$ eine metrische, also eine auswertbare Entsprechung verliehen bekommt. Im Spannungsfeld aktueller Fragestellungen aus den Bereichen Gebäude-, Energie- und Versorgungstechnik - im Allgemeinen - und in Bezug auf das hier behandelte Dimensionierungsproblem - im Speziellen - stellt das neue Kalkül eine sinnvolle Ergänzung zu den gängigen Methoden, die zur Behandlung von etwaigen Dimensionierungsproblemen herangezogen werden, bereit.

1. INTRODUCTION

In light of the ongoing transformation of energy supply, with the aim of making operations as resource-efficient and, where feasible, self-sufficient, new demands are being placed on energy management at the building and neighbourhood level. Meeting these demands depends on robust mathematical models of the underlying systems. However, the faithful modelling of operating assets - particularly heat engines and especially heat pumps - remains challenging: parameter variability, interacting subsystems, and context-dependent operating modes make high-precision models awkward and case-specific. This requires representations that balance physical accuracy with usability and remain portable across locations and use cases. In practice, models with sparse data must be calibratable yet sufficiently predictive to support technical decisions under a range of operating conditions.

To anticipate plant configuration for a specific application, a general yet simple approach is needed - one that captures the essential thermodynamic interactions and control behaviour without imposing detailed component realisations. Now the leitmotif here is to create a manageable modelling framework for heat pump sytems that couples an environmental model with interacting temperature zones to an observer of the compressor load, thus enabling a clear, criteria-based assessment of performance efficiency and control quality over a defined time intervals.

In practical terms, transparent assumptions and data economy are essential. Modelling is often constrained to time-invariant, deterministic, continuous dynamics with states expressed by temperatures and power flows, while available manufacturer data typically provide only sparse COP information. Under such conditions, interpolation schemes - e.g., NURBS-based completion of COP surfaces - serve to bridge missing regions and establish a consistent basis for simulation, validation, and comparative analysis without recourse to high-order component details.

2. STRUCTURE AND FUNCTIONING OF A HEAT PUMP

Meanwhile, the potential components of a future plant park can be technical systems of a rather complex nature, exhibiting highly non-linear system and response behaviour. As a consequence, setting up their dynamic models is correspondingly challenging. A heat pump, for example, consists - in general terms - of six main components. Briefly, and listed under their functional characteristics, these are:

1. **Evaporator:** Acts as a heat exchanger. At low temperatures and low pressure, a refrigerant evaporates, absorbing heat from the environment. If the absorbed heat is denoted by $\boldsymbol{u_1} \coloneqq \boldsymbol{Q}$ as the input variable, the state vector takes the form $\vec{\boldsymbol{x}}. \coloneqq [\boldsymbol{T} \quad \boldsymbol{m}]^{\boldsymbol{T}}$, where $\boldsymbol{x_2} \coloneqq \boldsymbol{m}$ denotes the mass of the refrigerant and $\boldsymbol{c}$ denotes its specific heat capacity, then for $\boldsymbol{x_1} \coloneqq \boldsymbol{T}$ as the temperature of the refrigerant in the evaporator, the heat transfer is described by the canonical relationship $\frac{\boldsymbol{dQ}}{\boldsymbol{dt}} = \boldsymbol{m} \cdot \boldsymbol{c} \cdot \frac{\boldsymbol{dT}}{\boldsymbol{dt}}$. Since the mass is constant, $\frac{\boldsymbol{dm}}{\boldsymbol{dt}} = \boldsymbol{0}$, and thus $\frac{\boldsymbol{dT}}{\boldsymbol{dt}} = \frac{\boldsymbol{u_1}}{\boldsymbol{x_2 \cdot c}}$. The dynamic behavior of the evaporator in state-space-representation form is given by the equation system: $\dot{\boldsymbol{x}} = \boldsymbol{A} \cdot \vec{\boldsymbol{x}} + \boldsymbol{b} \cdot \boldsymbol{u}, \;\; \boldsymbol{y} = \boldsymbol{c^T} \cdot \vec{\boldsymbol{x}}.$
2. **Compressor:** Increases the pressure and temperature of the refrigerant. The process can be described by the adiabatic compression equation $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$, where P_1 and V_1 denote the initial pressure and volume of the refrigerant, P_2 and V_2 the final pressure and volume after compression accordingly, with γ as the ratio of specific heats. The state space of the compressor is thus described by the vector $\vec{\boldsymbol{x}} \coloneqq [\boldsymbol{p} \quad \boldsymbol{T}]^{\boldsymbol{T}}$ and the input pressure $\boldsymbol{u_2} \coloneqq \boldsymbol{P}_{\text{in}}$. The system matrix A_2 results from a linearization of the corresponding differential equations of the form $\dot{\boldsymbol{x}} := \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) = [\boldsymbol{f_1}(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{u}) \quad \boldsymbol{f_2}(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{u})]^{\boldsymbol{T}}$. The two nonlinear (vector) functions $\boldsymbol{f_1}(\boldsymbol{x}, \boldsymbol{u})$ and $\boldsymbol{f_2}(\boldsymbol{x}, \boldsymbol{u})$ describe the compression behavior and depend on specific characteristics of the compressor used.
3. **Condenser:** In the condenser, the refrigerant releases the stored thermal energy while condensing through cooling or heat transfer. Without loss of generality, it may be assumed that the same relations hold for the condenser as for the evaporator. The state space vektor $\vec{\boldsymbol{x}} \coloneqq [\boldsymbol{T} \quad \boldsymbol{m}]^{\boldsymbol{T}}$ is likewise defined by the temperature and mass of the refrigerant. Only for the system input applies now $\boldsymbol{u_2} = \boldsymbol{Q}_{\text{out}} = -\boldsymbol{u_1}$, following the opposite sign convention, since it concerns the released heat $\boldsymbol{Q}_{\text{out}} = -\boldsymbol{Q}_{\text{in}}$, hence $\frac{\boldsymbol{dx_1}}{\boldsymbol{dt}} = \frac{\boldsymbol{-u_1}}{\boldsymbol{x_2 \cdot c}}$.

4. **Expansion Valve:** While the condenser, in a certain sense, represents the counterpart to the evaporator, the expansion valve or throttling element forms the corresponding complement to a compressor. After the refrigerant has passed through the condenser, the expansion valve reduces the pressure generated by the compressor, which in turn leads to a cooling of the refrigerant before it enters the evaporator again. The expansion process defines an isenthalpic state change and can be expressed by the relaxation equation $\boldsymbol{h_1} = \boldsymbol{h_2}$, where $\boldsymbol{h_1}$ and $\boldsymbol{h_2}$ denote the specific enthalpies before and after expansion, respectively. Analogous to the state space of the compressor, the state vector of the expansion valve is given as $\vec{\boldsymbol{x}} := [\boldsymbol{p} \quad \boldsymbol{T}]^{\boldsymbol{T}}$.

   Also in this case, no standardised description of the expansion valve can be given. Several aspects such as flow dynamics (flow rate, pressure losses, turbulence, cavitation), the thermodynamic properties of the refrigerant and the geometry of the valve can contribute to the complexity of these functions. As there are also different types of expansion valves (e.g. capillary tube expansion valves, throttle valves, etc.), which may differ between manufacturers, no universally valid and precise determination equation can be formulated here.

5. **Circulation System:** The connecting element of the components of the heat pump described so far is the circulation system itself. The refrigerant passes through a closed cycle, circulating between the different subsystems ⇒ compressor ⇒ condenser ⇒ expansion valve ⇒ evaporator ⇒ compressor ⇒ and so on. During the respective phases, the refrigerant is forced to change its state of aggregation from liquid to gaseous or from gaseous to liquid, whereby it either absorbs ambient heat or releases it back into the environment.

6. **Control Unit:** In addition to the classic components, modern heat pumps are also equipped with a control or even a regulation unit. This is used to the monitoring of relevant status and performance variables such as temperature, pressure, flow rate or other characteristic parameters of the refrigeration circuit. If suitable sensors are available, corrective measures can be taken in autonomous operation to maintain a predefined room temperature. Operating parameters such as evaporation and the condensation temperatures or the operating mode of the compressor can be adjusted to achieve this. Ultimately, the aim is to increase the performance efficiency of the heat pump.

3. DISCUSSION OF THE PROBLEM

The omnipresent demand for efficiency in modern energy systems requires, among other things, new methods of modelling. In many cases, the coupled sub-dynamics – load; plant; control - manifest in highly nonlinear response behaviour that is difficult to reproduce for a specific instance and even harder to generalise.

This is most evident in modern systems such as heat pumps, where component couplings, mode switching, and ambient dependencies lead to strongly context-dependent behaviour. Accordingly, a general yet simple modelling approach is required that captures the essential thermodynamic interactions and control effects without reliance on case-specific detail. For instance, in the context of heat-pump use, improved efficiency may be pursued through appropriate adjustment of operating parameters or through informed choices in th system dimensioning.

Suppose that all specifications and relevant indicators have been provided by the manufacturer for a specific version of a given heat pump type. For instance, the standard performance coefficients - ($\boldsymbol{COP}, \boldsymbol{EER}, \boldsymbol{SEER}$)- technical documentation on all operating modes, transparent disclosure of all control options, connections, interfaces and, where applicable, access rights at the inverter level. Furthermore, detailed characteristic curve recording and characteristic curves for varying load profiles and compressor loads with regard to all relevant state variables. Finally, the dynamic models of the above-mentioned components of a heat pump.

For a defined time interval, and given a sufficiently well-predicted course of external influencing factors such as ambient temperature or consumer behavior, it would be theoretically possible to achieve optimization in the sense of the efficiency improvement discussed here. This can be realized either through a system-theoretic analysis or by continuous adjustment of the control variables during operation, i.e., as part of a practical upgrading of the system. Such a system analysis can be carried out in two different manners: on the one hand in the time domain - through extensive simulation series of a detailed dynamic model - and on the other hand in the frequency domain - through the established means of modal analysis.

The fundamental issue with heat pumps is that, given variations in design, performance, and system behavior, the dynamics of each individual unit can only be accurately reproduced by adjusting general model parameters. In this sense, a manufacturer-independent universal model does not exist. Nevertheless, to derive reliable statements about how a pump must be configured to meet specific requirements, an appropriate invariant evaluation framework must first be established. Such a framework should enable the user, from a general perspective, to formulate precise conclusions regarding the design of the plant type, its sizing (maximum heating/cooling capacity), as well as its system and control behavior.

The overriding leitmotif is to find a general model for describing the state change behaviour of a heat pump that enables a qualitative assessment of both performance efficiency and control quality. To a certain extent, the task formulated in this way reflects the ambivalent nature of the problem at hand. Intuitively, one can already assume that performance efficiency and control behaviour are two opposing goals that - being interdependent - are in contradiction with each other.

Performance efficiency: The performance efficiency of a heat pump is the answer to the question of how efficiently heat is transferred between the source and the sink. The aim is to achieve maximum transfer with minimum power consumption from the supply network. In other words, it is a matter of operating the compressor with as little demand as possible. In case of doubt, high efficiency means slower control behaviour with correspondingly longer settling or integral times.

Control quality: The parameters used to describe control performance in the canonical sense are: rise time, overshoot, settling time and control deviation. Essentially, they characterise the heat pump in terms of how well it responds to setpoint changes or changes in the environment. A good performance will generally be understood to mean that the system's control device responds quickly to any changes, levels out setpoint deviations and maintains system stability. In an unfavourable case, for instance, in the case of significant, highfrequency load changes, this can lead to a frequent cycling on and off or abrupt changes of the heat pump's operating mode, which is likely to prevent an increase in performance efficiency.

Both of these objectives can be understood as functions of several variables. To assess the performance efficiency defined here, the coefficient of performance (COP) is suitable, expressed as the ratio of the heat output Q_{WP} to the electrical input power W_{el} required by the compressor. This is calculated using the quotient

$$\boldsymbol{COP} := \frac{\dot{\boldsymbol{Q}}_{\boldsymbol{wp}}}{\boldsymbol{W}_{\boldsymbol{el}}} \tag{1}$$

Under strongly simplifying assumptions, assuming an ideal reversible process, the COP can be expressed via the Carnot efficiency. Denoting by T_{out} and T_{in} the temperatures of the hot and cold reservoirs, respectively - i.e., the temperature of the brine at the heat sink (such as the separate water circulation system of a floor heating installation) and at the heat source (such as the outside air or the ground in geothermal systems) - the COP of an ideal heating heat pump is then given as:

$$\boldsymbol{COP} := \frac{\boldsymbol{T_{out}}}{\boldsymbol{T_{out}-T_{in}}} \tag{2}$$

However, the real COP value is usually significantly lower than the theoretical value. An accurate determination of the current COP is crucial for a reliable evaluation model. First, the implicit representation is assumed in general terms, as per:

$$\boldsymbol{\mathcal{F}(COP, T_{out}, T_{in}) = 0} \tag{3}$$

This expression will be refined in the course of this paper. It will be specially prepared and then given an explicit form in the form of a non-uniformly parameterised tensor product surface à la the general Bézier representation, according to:

$$\boldsymbol{\mathcal{M}(u, v) \coloneqq \sum_{i=0}^{n} \sum_{j=0}^{m} P_{ij} \cdot N_i(u) \cdot N_j(v)} \tag{4}$$

Control quality is influenced by several factors. On the one hand, by the controller type itself, as well as by the validation of the corresponding controller parameters. Furthermore, there are influences of load dynamics and the ambient temperature.

$$\boldsymbol{\mathcal{R} \coloneqq (L(t), \Delta T_{amb}, K_P, K_I, K_D, \dots)} \tag{5}$$

The use of a cost function is useful in evaluating the control quality here. For example, in the form of the ITAE criterion (integral of time multiplied by the absolute value of error) according to Kulebakin-Mandelstamm or its modification.

$$\boldsymbol{J = \int_0^\infty t \cdot | e(t) | \, dt} \tag{6}$$

Here, $e(t)$denotes the steady-state control error. In this case, it is defined as the difference between the predicted room temperature $T_z(t)$and the setpoint T_{set}:

$$\boldsymbol{e(t) \coloneqq T_{set} - T_z(t)} \quad [K] \tag{7}$$

Assuming that the thermostat is operating in the form of a canonical PID controller, then the heating power $\dot{Q}_{wp}^{reg}$ at the controller output can be given as follows:

$$\boldsymbol{\dot{Q}_{wp}^{reg} = K_P \cdot e(t) + K_I \int e(t)\, dt + K_D \frac{de(t)}{dt}} \quad [W \equiv Js^{-1}] \tag{8}$$

In general, the thermal energy necessary to increase the temperature of a mass $\boldsymbol{m}[kg]$ with thespecific heat capacity $\boldsymbol{c}\left[\frac{J}{kg \cdot K}\right]$ by $\boldsymbol{\Delta T}$ $[K]$ is given by the relation:

$$\boldsymbol{Q = m \cdot c \cdot \Delta T} \; [\,\mathrm{J}\,] \tag{9}$$

Furthermore, with a heat transfer coefficient $\alpha\left[\frac{W}{m^2 \cdot K}\right]$and a contact area $A[m^2]$, Fourier's law applies to the convective heat transfer rate, which results as follows:

$$\frac{dQ}{dt} \equiv \dot{Q} = \alpha \cdot A \cdot \Delta T \qquad [W \equiv Js^{-1}] \tag{10}$$

Finally, denoting the fluid density by $\boldsymbol{\rho}\left[\frac{kg}{m^3}\right]$, the specific heat capacity of the fluid at constant pressure by $\boldsymbol{c_p}\left[\frac{J}{kg \cdot K}\right]$, the corresponding control volume by $\boldsymbol{V}[m^3]$, and the heat power supplied to or removed from this volume by $\dot{\boldsymbol{Q}}[W]$, its energy balance can be expressed as follows, simply by relating heat flow to temperature:

$$\frac{dT}{dt} \equiv \dot{T} = \frac{1}{c_p \cdot \rho \cdot V} \cdot \dot{Q} \quad \left[\frac{T}{s}\right] \tag{11}$$

Based on this, a dynamic environment and observer model will be created below.

4. MODEL DEVELOPMENT

In light of the difficulties discussed at the outset in modelling thermal engines, we will now refrain from simulating the system behaviour of the heat pump using a detailed dynamic model. This approach does eliminate the possibility of examining the ‘fine’ structural dynamics of the system at higher resolution time scales, but as already mentioned, such an examination is only necessary if specific questions about a specific type of system need to be answered. In other cases, a balance must be struck between the model complexity and the practicality of its fulfilment.

The analysis of the pump dynamics here is accomplished using two models, both of which are applied simultaneously. Based on the relationships (8)–(11), the first is an environment model configured from several interconnected temperature zones, thus mapping the interactions of the heat exchange system. Depending on the resulting temperature differences between individual zones, it determines the corresponding transfer rates and also investigates the response behaviour as a result of an external intervention through the operation of controls and disturbances.

The second is an observer model, which uses equations (1)–(8) to determine the compressor demand and qualifies the performance of the heat pump over any time interval for a defined load profile. The controller output thus represents the coupling element between the first and second model. However, the control model in (8) is not mandatory and can be designed or parameterised in many different ways.

5. ENVIRONMENT MODEL

By means of the energy balances established above, the present environment model is formulated as a deterministic, nonlinear, time-invariant, dynamic, continuous, and coupled multi-variable system. In state-space $\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}$ with output $\boldsymbol{y} = \boldsymbol{Cx} + \boldsymbol{Du}$, it becomes a differential equation system of order n, where

$$\frac{dT_i}{dt} = \frac{1}{\rho c_p V_i}\left(\sum_{j\neq i}^{n} \alpha_{ij} A_{ij}\left(T_j - T_i\right) + Q_{\text{ext},i}\right) \tag{12}$$

represents a generalized expression that describes the thermodynamics of the i-th of a total of n −coupled temperature zones of a heat transfer system. For the formulation of this system of differential equations, the signal flow diagram sketched in Fig. 1 is employed. The following notations and conventions are adopted:

T_V − SupplyTemperature; T_R − ReturnTemperature; T_U − AmbientTemperature;
T_Z − RoomTemperature; T_B − FloorTemperature; T_E − Ground Temperature;
T_r − Thermostat Temperature; W_{el} − Electrical Power; $\dot{Q}_E$ − Geothermal Power;
$\dot{Q}_{SB}$ − Heat pump Power; $\dot{Q}_{SB}$ − Floor heat Flow; $\dot{Q}_{BZ}$ − Room heat Flow;
$\dot{Q}_{diss}$ − Dissipation Power; V − Control Volume; A − Floor Contact Area;
ρ − Fluid density; m − Fluid mass; α_{SB} − Heat transfer coefficient oil-floor;
α_{BZ} − Heat transfer coefficient floor-room; c_p − Specific heat Capacity;

The state space of interest here is defined by the vector $\underline{X} := [T_V \;\; T_R \;\; T_B \;\; T_Z]^T$. The power flows shown in figure below are entered according to the principle of completeness. However, certain simplifying assumptions can be introduced in this case. For example, geothermal power $\dot{\boldsymbol{Q}}_E$ and dissipation power $\dot{\boldsymbol{Q}}_{\boldsymbol{diss}}$ can be combined into a single external power input $\dot{\boldsymbol{Q}}_{\boldsymbol{A}}$. Accordingly, $\boldsymbol{T_E}$ and $\boldsymbol{T_U}$ can also be combined into an external temperature $\boldsymbol{T_A}$. Finally, the heat transfer rates $\dot{\boldsymbol{Q}}_{\boldsymbol{SB}}$ and $\dot{\boldsymbol{Q}}_{\boldsymbol{BZ}}$ can be combined and treated together with the pump power $\dot{\boldsymbol{Q}}_{\boldsymbol{WP}}$.

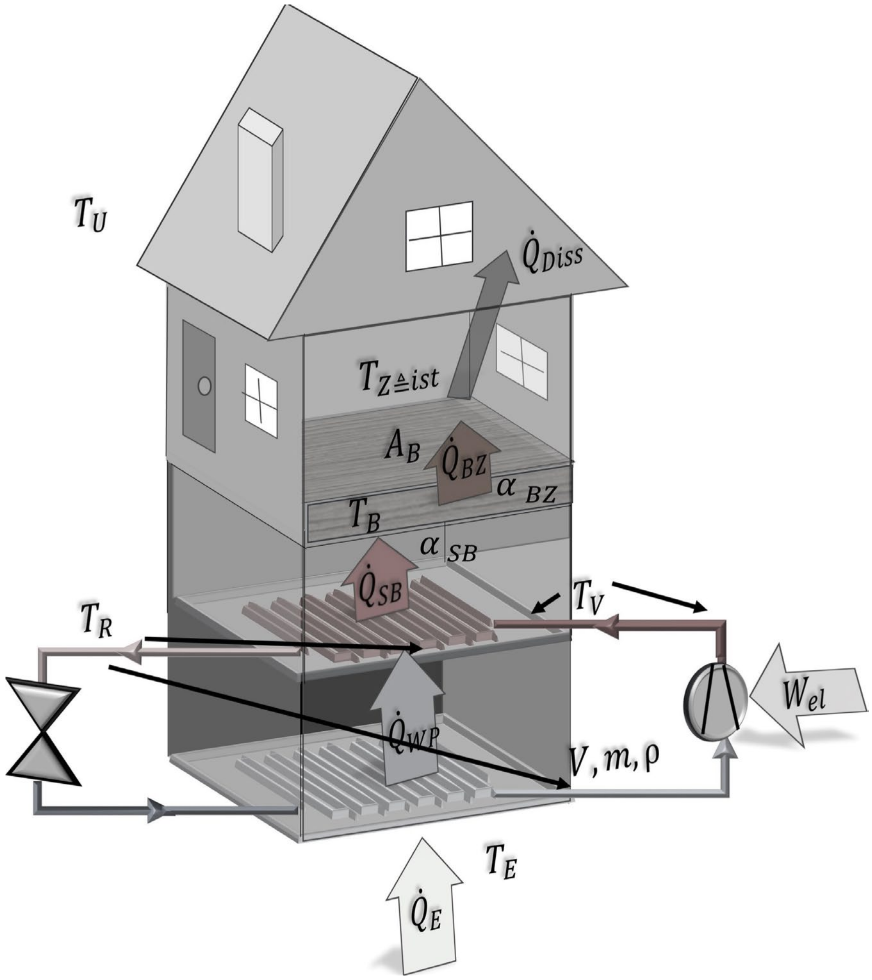

Fig. 1: Functional principle and signal flow diagram of a heat pump

The unit contained in the brackets of equation (12) is $[\mathrm{W} \equiv \mathrm{J} \cdot \mathrm{s}^{-1}]$, and accordingly $[\mathrm{J} \cdot \mathrm{K}^{-1}]$ outside the bracket. Furthermore, for each index pair $\boldsymbol{i}, \boldsymbol{j}$ where $\boldsymbol{i} \neq \boldsymbol{j}$ the coefficients are defined by the expressions: $\boldsymbol{\mathcal{K}_{ij}} \coloneqq \frac{\alpha_{ij} A_{ij}}{\rho c_p V_i}$, $\boldsymbol{\kappa_i} \coloneqq \frac{-1}{\rho c_p V_i}$.

Thus, the system is of fourth order. The power balance of the supply temperature can be directly derived from the figure. The heat pump power $\dot{\boldsymbol{Q}}_{\boldsymbol{WP}}$ here acts as a system input and must be applied directly to the state variable $\dot{X}_v$ via the control matrix. Furthermore, - caused by a developing gradient $\boldsymbol{\Delta T_{V,R}} \coloneqq (\boldsymbol{T_V} - \boldsymbol{T_R})$ - heat transfer occurs through convection. If $\mathcal{K}_{\boldsymbol{12}}(\boldsymbol{\Delta T_{V,R}}) > \boldsymbol{0}$, this applies in the direction of the return flow. Otherwise, the process can be interpreted as cooling. Finally, the rate of change of the supply temperature $\boldsymbol{T_V}$ is obtained the following:

$$\frac{d\boldsymbol{T_V}}{d\boldsymbol{t}} = \mathcal{K}_{\boldsymbol{V,R}} \cdot (\boldsymbol{T_V} - \boldsymbol{T_R}) + \kappa_V \cdot \dot{\boldsymbol{Q}}_{\boldsymbol{wp}} \tag{13}$$

The return temperature is again not directly influenced by a system input. Its rate of change results solely from convection due to the temperature gradient toward the adjacent neighboring zones. With the corresponding sign inversion, it is initially the first summand from (13). In addition to this, a temperature gradient $\Delta \boldsymbol{T_{R,B}} \coloneqq (\boldsymbol{T_R} - \boldsymbol{T_B})$ is introduced, representing the difference between the return temperature $\boldsymbol{T_R}$ and the floor temperature $\boldsymbol{T_B}$. Summing up, the balance of $\dot{\boldsymbol{T}}_{\boldsymbol{R}}$ is:

$$\frac{d\boldsymbol{T_R}}{dt} = -\mathcal{K}_{\boldsymbol{V,R}} \cdot (\boldsymbol{T_V} - \boldsymbol{T_R}) + \mathcal{K}_{\boldsymbol{R,B}} \cdot (\boldsymbol{T_R} - \boldsymbol{T_B}) \tag{14}$$

In a similar way, the thermodynamics of the floor is modeled. With the convention $\mathcal{K}_{\boldsymbol{32}} \coloneqq -\mathcal{K}_{\boldsymbol{23}}$ and a temperature gradient relative to the room temperature - i.e., the actual system output $\boldsymbol{y} \coloneqq \boldsymbol{T_z}$ – the third differential equation is then given by:

$$\frac{d\boldsymbol{T_B}}{dt} = -\mathcal{K}_{\boldsymbol{R,B}} \cdot (\boldsymbol{T_R} - \boldsymbol{T_B}) + \mathcal{K}_{\boldsymbol{B,Z}} \cdot (\boldsymbol{T_B} - \boldsymbol{T_Z}) \tag{15}$$

Finally, the dynamics of the room temperature can also be formulated in the same way. In addition to the convection of the floor heat with $\mathcal{K}_{\boldsymbol{ZB}} \coloneqq -\mathcal{K}_{\boldsymbol{BZ}}$, a second input is imposed on the overall system. This is expressed via the temperature gradient to the ambient environment $\boldsymbol{\Delta T_{ZA}} \coloneqq (\boldsymbol{T_Z} - \boldsymbol{T_A})$. The final equation is thus:

$$\frac{d\boldsymbol{T_Z}}{dt} = -\mathcal{K}_{\boldsymbol{BZ}} \cdot (\boldsymbol{T_B} - \boldsymbol{T_Z}) + \mathcal{K}_{\boldsymbol{ZA}} \cdot (\boldsymbol{T_Z} - \boldsymbol{T_A}) \tag{16}$$

The overall heat transfer system can be written in a canonical state space notation:

$$\begin{bmatrix} \dot{\boldsymbol{T}}_{\boldsymbol{V}} \\ \dot{\boldsymbol{T}}_{\boldsymbol{R}} \\ \dot{\boldsymbol{T}}_{\boldsymbol{B}} \\ \dot{\boldsymbol{T}}_{\boldsymbol{Z}} \end{bmatrix} = \underline{\underline{\boldsymbol{A}}}^{\mathbf{4x4}} \cdot \begin{bmatrix} \boldsymbol{T_V} \\ \boldsymbol{T_R} \\ \boldsymbol{T_B} \\ \boldsymbol{T_Z} \end{bmatrix} + \underline{\underline{\boldsymbol{B}}}^{\mathbf{4x2}} \cdot \begin{bmatrix} \dot{\boldsymbol{Q}}_{\boldsymbol{WP}} \\ \boldsymbol{T_A} \end{bmatrix} \tag{17}$$

with the system matrix $\underline{\underline{\boldsymbol{A}}}^{\mathbf{4x4}} = \begin{bmatrix} -\mathcal{K}_{VR} & \mathcal{K}_{VR} & \mathbf{0} & \mathbf{0} \\ \mathcal{K}_{VR} & -(\mathcal{K}_{VR} + \mathcal{K}_{RB}) & \mathcal{K}_{RB} & \mathbf{0} \\ \mathbf{0} & \mathcal{K}_{RB} & -(\mathcal{K}_{VR} + \mathcal{K}_{RB}) & \mathcal{K}_{BZ} \\ \mathbf{0} & \mathbf{0} & -\mathcal{K}_{BZ} & (\mathcal{K}_{BZ} + \mathcal{K}_{ZA}) \end{bmatrix}$

control matrix $\underline{\underline{\boldsymbol{B}}}^{\mathbf{4x2}} = \begin{bmatrix} \boldsymbol{\kappa_V} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathcal{K}_{ZA} \end{bmatrix}$ and the output signal $\boldsymbol{y} = [\mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{1}] \cdot \begin{bmatrix} \boldsymbol{T_V} \\ \boldsymbol{T_R} \\ \boldsymbol{T_B} \\ \boldsymbol{T_Z} \end{bmatrix}$

6. OBSERVER MODEL

The guiding idea of the observer model is that, during a time-series simulation, an estimation algorithm runs simultaneously and continuously in parallel with the environment model. The electrical drive power $\hat{\boldsymbol{x}} \coloneqq \boldsymbol{W_{el}}$ is the quantity to be estimated. In operation, the compressor load is thus to be quantitatively assessed, which is expected to reveal additional potential opportunities for an optimization.

The observer isn't designed in the canonical way. A classic Luenberger observer:

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{L}(\boldsymbol{y} - \boldsymbol{C}\hat{\boldsymbol{x}}) \tag{18}$$

with the estimated state vector $\hat{x}$ and observer matrix $\boldsymbol{L}$, is not employed. Instead, information is obtained by evaluating a special solution space defined as follows

$$\mathbb{L} \coloneqq \{\, \boldsymbol{P} \coloneqq (\mathrm{COP}, \boldsymbol{T_{out}}, \boldsymbol{T_{in}}) \in \mathbb{R}^3 \mid \boldsymbol{\mathcal{F}}(\boldsymbol{P}) = \boldsymbol{0} \,\} \tag{19}$$

Thus, the solution space $\mathbb{L}$ comprises all ordered triples $(\boldsymbol{COP}, \boldsymbol{T}_{\mathbf{out}}, \boldsymbol{T}_{\mathbf{in}})$ which, when satisfying $\boldsymbol{\mathcal{F}}(\boldsymbol{P}) = \boldsymbol{0}$, form elements of the solution set and can be interpreted as points on a parameter surface $\boldsymbol{P} \in \boldsymbol{\mathcal{M}}(\boldsymbol{u}, \boldsymbol{v}) \subseteq \mathbb{L}$. Each temperature pair $(\boldsymbol{T}_{\mathbf{out}\,=\,\boldsymbol{v}}, \boldsymbol{T}_{\mathbf{in}\,=\,\boldsymbol{u}})$ is uniquely associated with a corresponding performance value $\boldsymbol{COP}(\boldsymbol{T_v}, \boldsymbol{T_u})$. However, no explicit representation of such a parameter surface is available a priori. Nevertheless, in order to obtain the intended performance evaluation, the solution set $\mathbb{L}$ in (19) has to be either computable or already available.

In most cases, however, manufacturer specifications are limited to performance curves for selected temperature ranges, typically presented as measurement profiles from internal qualification tests. A fully defined solution space with tabulated values for every operating pair - i.e., a COP specification for all $(\boldsymbol{T_v}, \boldsymbol{T_a}) \in \mathbb{R}^3$ satisfying condition (19) - will, for obvious reasons, not be found in any manual.

One approach to completing a "sparse" dataset is through Bézier-based spline inter-polation. Its generalized form is known as non-uniform rational B-splines, or simply NURBS. Here, non-uniformity refers to the uneven distribution of knots within a knot vector, causing the basis functions to influence different areas of the spline to varying degrees. This makes it possible to adjust existing control points in a more differentiated manner, thereby achieving a higher precision in shaping.

A. ANNOTATION ON THE TECHNICAL BACKGROUND

Without going into further detail on the rather complex procedures involved in spline formation and numerical modelling of free-form surfaces, this section will be restricted to summarising an overview of the key terminology and principles.

Basis Functions: These represent a fundamental element of spline modeling. Basis functions serve, in a sense, as carriers of information. Superimposed basis functions provide the framework and determine the shape. They are polynomial constructs used to control the weighting of control points and thereby generate the desired curve form. Their key properties are locality and continuity at the transitions between the adjacent segments, enabling a continuously closed curve design.

$$\boldsymbol{N_{i,p}(u) = \frac{u-u_i}{u_{i+p}-u_i} \cdot N_{i,p-1}(u) + \frac{u_{i+p+1}-u}{u_{i+p+1}-u_{i+1}} \cdot N_{i+1,p-1}(u)} \quad (20)$$

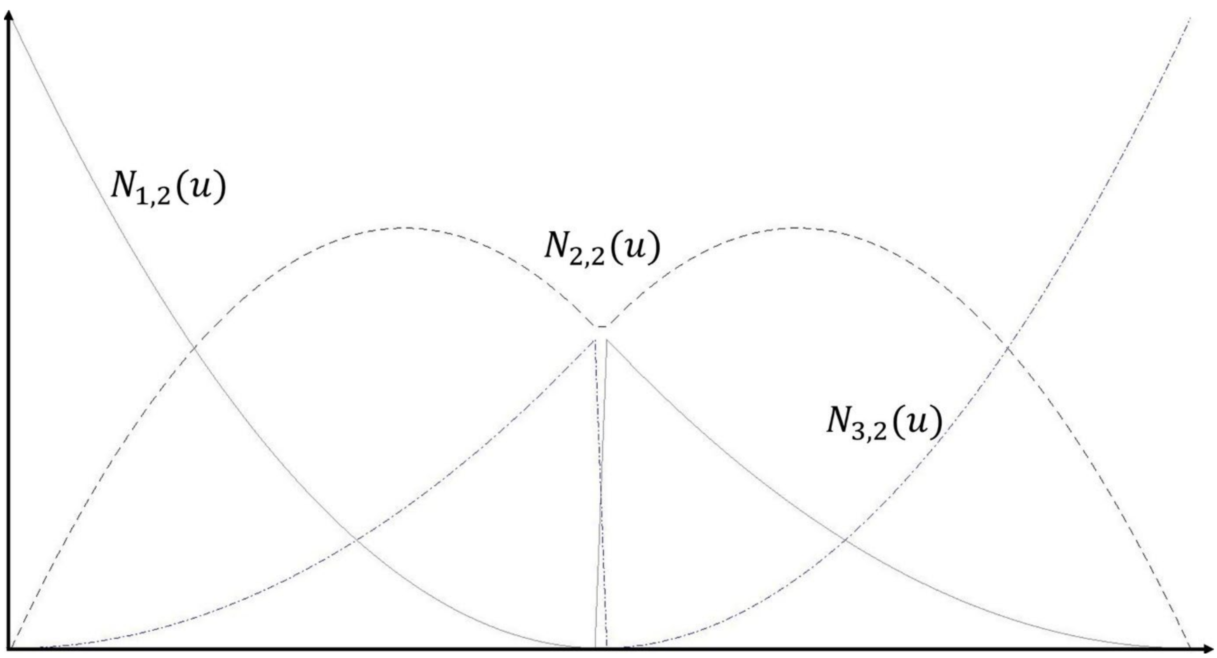

Fig. 2: Basis functions of non-uniform B-splines of second order

Here, $p + 1$ denotes the spline order. Otherwise, the partition of unity holds for each subinterval $u \in [u_i, u_{i+1})$. This property ensures that the curve is formed as a weighted sum of basis functions, providing stability and smooth shape control.

Knot Vectors are ordered sequences of knots. Knots are parameter values that define transitions from one basis function to the next. They influence the subdivision and number of basis functions along the spline. They define the domains for the basis functions and specify how these are allocated within a parameter space.

$$\boldsymbol{U} = \{ \underbrace{\boldsymbol{u_0}, \dots, \boldsymbol{u_p}}_{p+1}, \boldsymbol{u_{p+1}}, \dots, \boldsymbol{u_{r-p-1}}, \underbrace{\boldsymbol{u_{r-p}}, \dots, \boldsymbol{u_r}}_{p+1} \} \tag{21}$$

Control Points: The control points are the fundamental "building blocks" of a spline. They are elements of the solution space and define either curve segments $P_i \in \mathbb{R}^2$ for $u \in [0,1]$, or a discrete grid $P_{i,j} \in \mathbb{R}^3$ for $u \in [0,1]$ and $v \in [0,1]$. Modifications to their positions will change the shape of the curve or the surface.

$$\boldsymbol{P} = \{\boldsymbol{P_0}, \boldsymbol{P_1}, \boldsymbol{P_2}, \dots, \boldsymbol{P_n}\}, \quad \boldsymbol{P_i} \in \mathbb{R}^3 \tag{22}$$

Weights: These are shape factors assigned to each control point of a spline or surface point $P \in S(u)$. They determine the degree to which each control point influences the curve or the surface. A higher weight increases the influence of the corresponding control point, while a lower weight reduces its effect accordingly.

$$\boldsymbol{W} = \{\boldsymbol{w_0}, \boldsymbol{w_1}, \boldsymbol{w_2}, \dots, \boldsymbol{w_n}\}, \qquad \boldsymbol{w_i} \in \mathbb{R} \tag{23}$$

B. DESIGN OF AN ASSOCIATED SOLUTION SPACE

Now it is the task to construct an equivalent solution set. Let us assume that a heat pump manufacturer's technical manual provides information on how a heat pump works. This includes performance characteristics. In accordance with the illustration below, these characteristics exemplify the operating behaviour of the system based on recorded measurement curves for a few selected ranges of temperatures.

The characteristics are synthetic but may nonetheless correspond to the profiles of commercially available heat pumps. The polygonal curves plotted against the ambi-ent temperature T_A illustrate performance variations $COP(T_V, T_A)$ at constant suply temperatures $T_V = const$. The measurement points marked with □ define a control point set $P = \{P_0, P_1, \dots, P_n\}$ in the sense defined above, as per (22).

A non-uniform NURBS extrapolation across a 2D parameter space results in a tensor product surface. The defining relation behind this construction is given by the generalised expression in (4) and is standard in geometric modelling.

$$\boldsymbol{\mathcal{M}}(\mathbf{u}, \mathbf{v}) = \frac{\sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) w_{ij} P_{ij}}{\sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) w_{ij}} \tag{24}$$

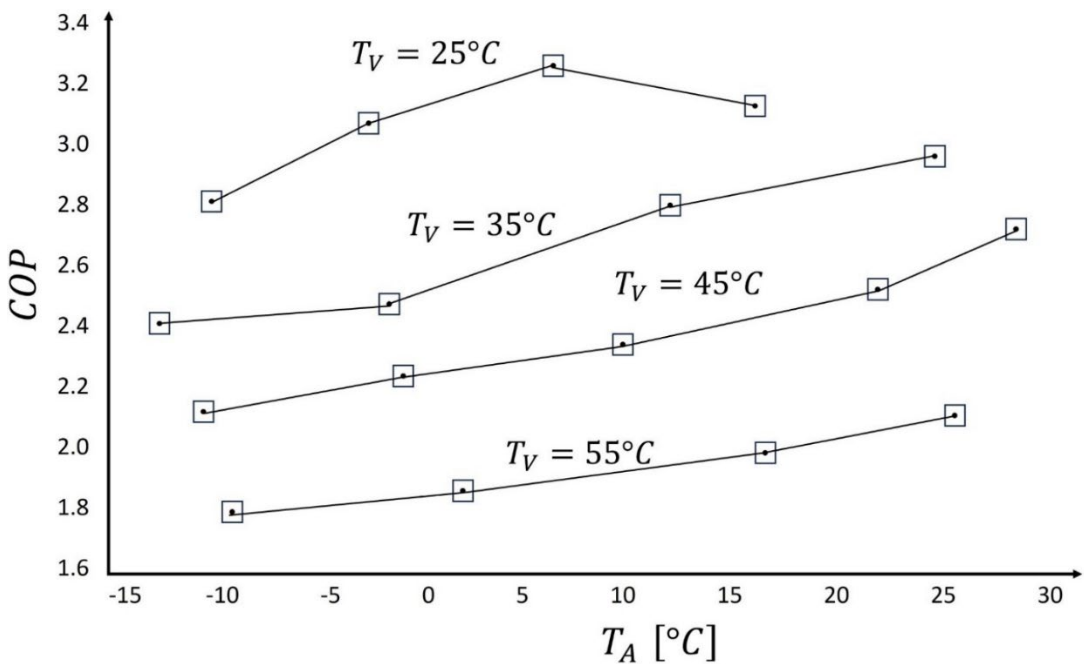

Fig. 3: Synthetic profile of the coefficient of performance (COP)

The weighting set according to (23) is initially defined for $n$control points as a unit vector: $W := \mathbf{e} = [1,1,\dots,1]_{n\times 1}$. The extrapolation proceeds in both directions of the scaled parameter space $\boldsymbol{u} \in [\mathbf{0},\mathbf{1}]$ and $\boldsymbol{v} \in [\mathbf{0},\mathbf{1}]$, each with order $\boldsymbol{p} = \boldsymbol{q} = \mathbf{2}$, and is assigned the knot vectors U=[0 0 0 0.5 1 1 1], V=[0 0 0 0.5 1 1 1]. When an appropriate routine is implemented and executed in program code, a product surface emerges, which can be interpreted and used as the solution space of a pump. The underlying principle will now be shown using a concrete example.

7. ON THE GUIDING APPROACH

Over a defined time interval $\boldsymbol{I_t} := [\boldsymbol{t_0}, \boldsymbol{t_{end}}]$, the environment model assumes different states during operation. Depending on the ambient temperature $\boldsymbol{T_A}(\boldsymbol{t})$ and the load profile $\dot{\boldsymbol{Q}}_{\boldsymbol{wp}}^{\boldsymbol{reg}}(\boldsymbol{t})$, a distinct state trajectory $\mathbf{X}(\boldsymbol{t}) \in \mathbb{R}^{\mathbf{1\times 4}}$, $\forall \boldsymbol{t} \in \boldsymbol{I_t}$, emerges in each case. Each state $\mathbf{X}^{\boldsymbol{T}} = [\,\boldsymbol{T_v}\ \ \boldsymbol{T_R}\ \ \boldsymbol{T_B}\ \ \boldsymbol{T_Z}\,]$ is a solution of the differential-algebraic system (17), or equivalently, of the environment model represented therein. For a given pump type, every state trajectory is therefore equivalent to a solution curve. This, in turn, corresponds to a trajectory $\boldsymbol{\gamma}: \boldsymbol{I_t} \subset \mathbb{R} \to \mathbb{R}^{\mathbf{3}}$ that satisfies condition given in (19) $\forall\, \boldsymbol{t} \in \boldsymbol{I_t}$, thus embedded in the solution space $\mathbb{L}$.

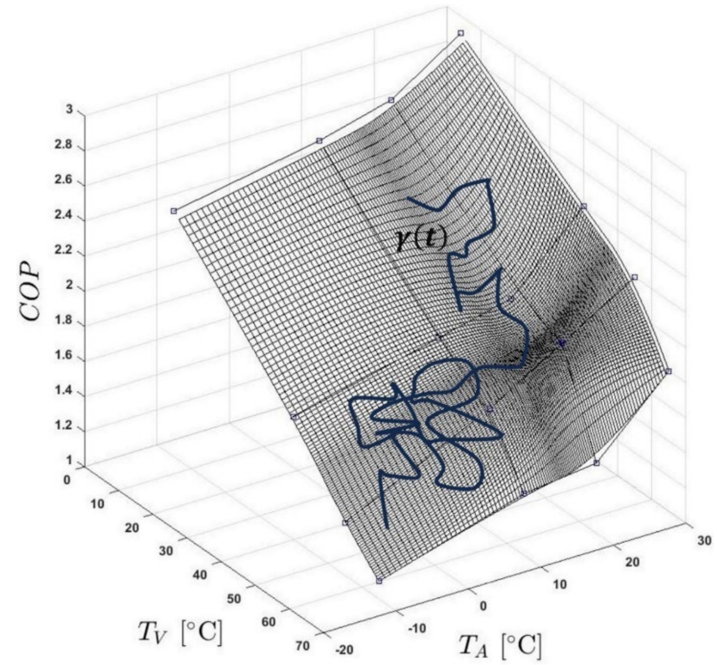

Fig. 4: Tensor product surface as the associated solution space of a pump

Now it is also clear why the tensor product space constructed in the previous section was referred to as a solution space. $\boldsymbol{\mathcal{F}(P)} = \mathbf{0}$ or, equivalently, $\boldsymbol{\mathcal{M}(u, v)} \in \mathbb{R}^{\mathbf{3}}$, constitutes a solution space associated with the environment model (13)-(18).

However, the observer model is not yet complete. It should be remembered that the guiding principle is to use the observer model to evaluate plant operation so that optimisation can be achieved on this basis. Two prerequisites have to be satisfied in order to achieve this. On the one hand, corresponding efficiency criteria have to be defined. On the other hand, an evaluation calculation must be available.

It should be noted(!) that the two models - the environment model and the observer model - are defined in different spaces. They are parameterized differently. While the observer model is constructed in Euclidean $\mathbb{R}^3$, the environment model initially exists in a space endowed with a different metric. In solving the system of differential equations, each state $\mathbf{X}^{\boldsymbol{T}} = [\boldsymbol{T_v}\ \boldsymbol{T_R}\boldsymbol{T}\ _{\boldsymbol{B}}\ \boldsymbol{T_Z}]$ is mapped into Euclidean $\mathbb{R}^4$. Despite of this, the designed solution space of the environment model continues to represent a discrete mesh and is therefore not a Riemannian manifold.

In order to evaluate a temperature pair within the associated solution space (19) $\mathbb{L} = \{(\boldsymbol{COP}, \boldsymbol{T_{out}}, \boldsymbol{T_{in}}) \in \mathbb{R}^{\mathbf{3}} \mid \boldsymbol{\mathcal{F}(P)} = \mathbf{0}\}$, a suitable mapping $\boldsymbol{g_P}: (\boldsymbol{T_V}, \boldsymbol{T_A}) \rightarrow (\boldsymbol{T_{out}}, \boldsymbol{T_{in}})$ must first be defined. Strictly speaking, the associated solution space is endowed with a metric via $\boldsymbol{g_P}: \boldsymbol{T_p\mathcal{M}} \times \boldsymbol{T_p\mathcal{M}} \rightarrow \mathbb{R}$, thereby **regularizing** existing definitional gaps. Here, $\boldsymbol{T_p\mathcal{M}}$ denotes the tangent space at point $\boldsymbol{p} \in \boldsymbol{\mathcal{M}}$.

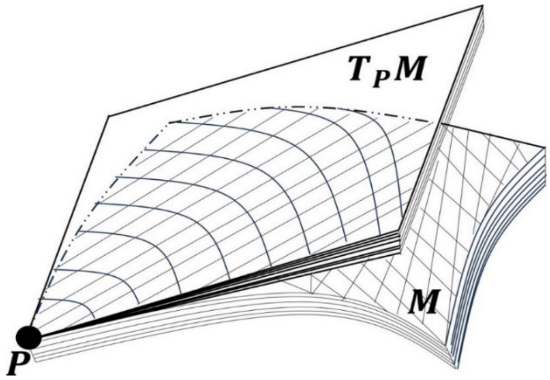

Fig. 5: Tangent space $T_p\mathcal{M}$ at a point $p \in \mathcal{M}$

From a technical perspective, this means that at every defined point of the grid a scalar product of the form $\boldsymbol{g_p(X,Y) = \langle X,Y\rangle = X_1Y_1 + X_2Y_2}$ is carried out, thereby spanning tangent spaces across $\boldsymbol{\mathcal{M}}$. Here, $\boldsymbol{X}$ and $\boldsymbol{Y}$ are vectors in $\mathbb{R}^2$, i.e., $\boldsymbol{X = (X_1, X_2)} \;\widehat{=}\; \boldsymbol{(T_A, T_{in})}$ and $\boldsymbol{Y = (Y_1, Y_2)} \;\widehat{=}\; \boldsymbol{(T_V, T_{out})}$**.** The scalar product defined via the Riemannian metric $\boldsymbol{g_p(X,Y)}$ induces a distance function of form

$$\boldsymbol{d(COP_{out,in}, COP_{V,A}) = (T_{out} - T_V)^2 + (T_{in} - T_A)^2} \tag{25}$$

Further, the distance function $\boldsymbol{d}$ between two points $\boldsymbol{P_1}$ and $\boldsymbol{P_2}$ on a Riemannian manifold is defined as the infimum of the lengths of all piecewise differentiable curves from $\boldsymbol{P_1}$ to $\boldsymbol{P_2}$. With $\boldsymbol{\gamma: [0,1] \to \mathcal{M}, \gamma(0) = P_1,, \gamma(1) = P_2}$ one obtains

$$\boldsymbol{d(P_1, P_2) = \inf\left\{\int_0^1 \sqrt{g(\dot{\gamma}(t), \dot{\gamma}(t))}\, dt\right\}} \tag{26}$$

In this way, for each given temperature value pair $(\boldsymbol{T_V}, T_A)$, the associated triplet $\boldsymbol{P := (COP,\ T_{out},\ T_{in})}$ on the mesh grid $\boldsymbol{\mathcal{M}(u,v)}$ of the observer model is determined, and thus the demanded performance index can ultimately be obtained.

This elementary, yet rather elegant sleight of hand now permits the simultaneous determination of all required key quantities: on the one hand, the state vector of the actual process — i.e., the environment model — $\boldsymbol{\mathrm{X}^T = [T_v\ \ T_R T_B T_Z]}$ and on the other hand, the performance index $\boldsymbol{COP(T_{out}, T_{in})}$as provided by the observer model. Furthermore, with a prescribed reference temperature $\boldsymbol{T_{set}}$**,** the system input of the closed control loop is given in the form of the heating power $\boldsymbol{u := \dot{Q}_{WP}^{reg}}$.

Finally, using the defining equation from (1), one can determine the drive power $\boldsymbol{W_{el} = \dot{Q}_{WP}^{reg} \cdot COP(T_{out}, T_{in})^{-1}}$ which enables for a quantitative assessment of the compressor load. With this, the evaluation framework is established, enabling the definition of quality criteria and a discussion of potential optimization options.

8. PREPARATION OF A SIMULATION ENVIRONMENT

Testing of the complete model - *environment model and observer model* - under varying load profiles, imposed disturbance signals, different settings of controller dynamics, and other switching actions can now help to identify and exploit potential improvements. The figure below shows the circuit diagram or signal flowchart of the overall model. It consists of three substructures. In addition to the environment and observer blocks, $\mathcal{U}$ and $\mathcal{B}$ respectively, it also includes the controller $\mathcal{R}$. In accordance with eq. (8), the latter is initially implemented as a PID element.

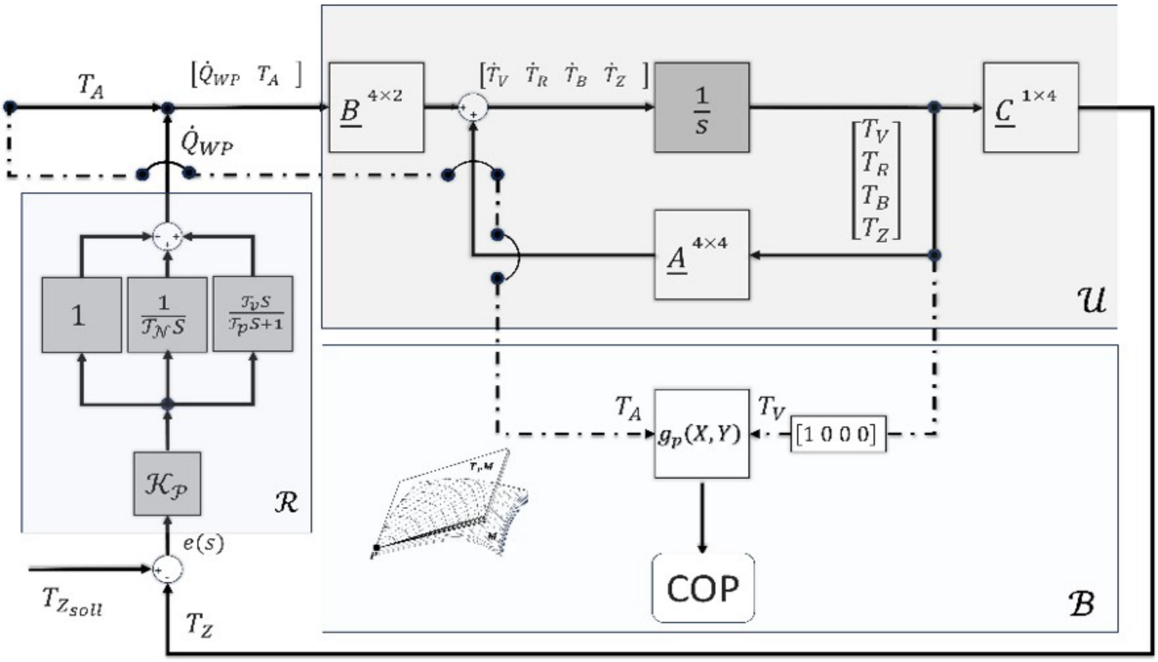

Fig. 6: Signal flowchart of the time-series model

For a period of one year, synthetic profiles are assumed for the ambient temperature $\boldsymbol{T_A(t)}$ and load demand profile $\boldsymbol{T_{Z_{soll}}(t)}$. The equidistant resolution is 1 h.

It is further agreed that the heat pump is considered solely as a system for space heating. According to Fig. 8, the reference trajectory of the load demand is constructed accordingly. The setpoint temperature determined by the thermostat extends, over the course of operation, across the temperature range: $T_{Z_{soll}}(t) := \{\ [\ 18°C,\ 25°C\ ] \cup \{\ 0°C\ \}\ \forall\ t\ \in I_t \backslash I_{t3}\ \}$ with the annua timel interval $I_t := [t_0 \equiv 0, t_{end} \equiv 8760\ \text{h}]$ and summer period $I_{t3} = [3600\ \text{h}, 5760\ \text{h}]$.

So this means that the thermostat is set to $T_{Z_{soll}} = 0°C$ during the third quarter.

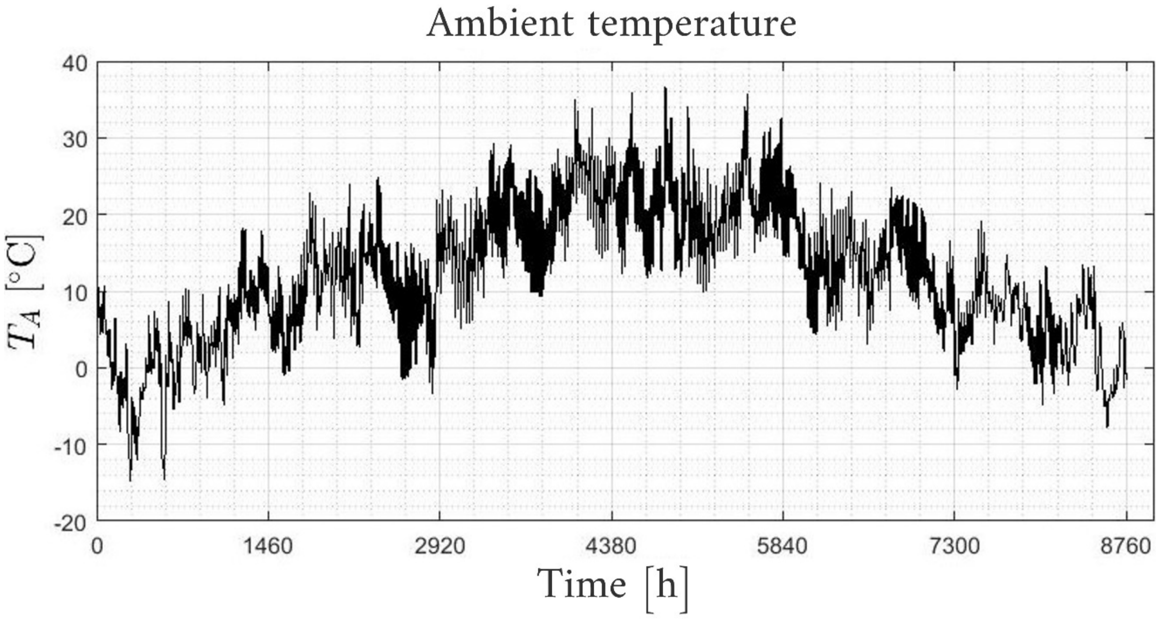

Fig. 7: Synthetic annual profile of the ambient temperature $\boldsymbol{T_A(t)}$

The characteristic values for convective transfer rates, density as well as control volume and mass of the fluid, the specific heat capacities, and the contact area are synthetic relative modeling assumptions. They constitute coefficients of thesys-tem and control matrices $\boldsymbol{A^{4\times4}}$ and $\boldsymbol{B^{4\times2}}$ according to (17) and (18), respectively.

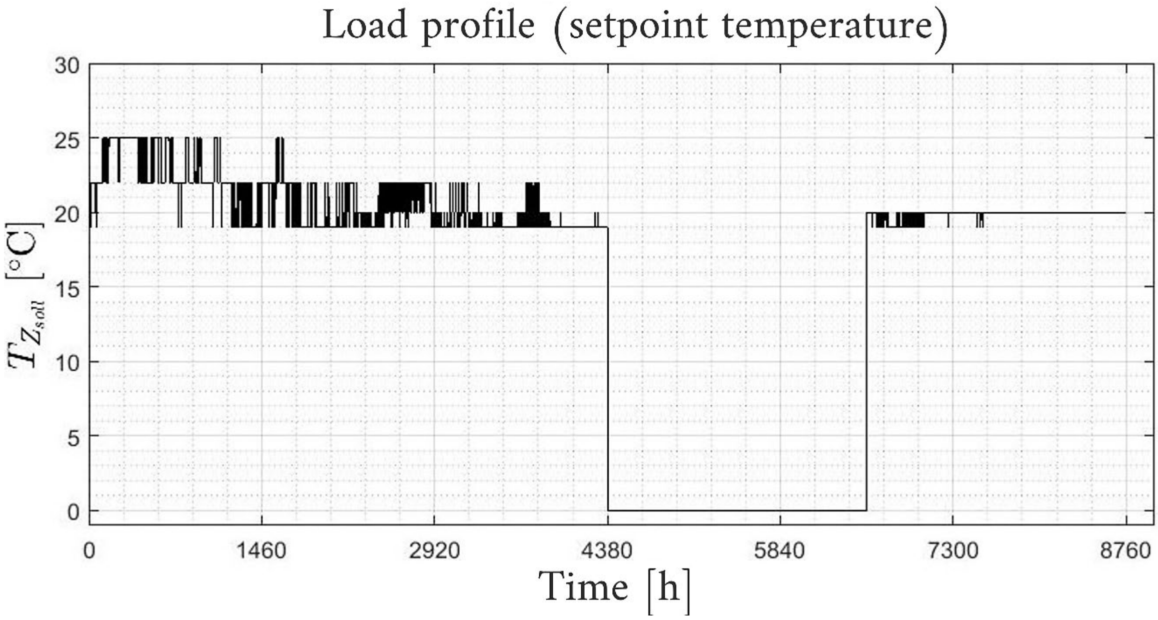

Fig. 8: Synthetic curve of a load profile dynamic $\boldsymbol{T_{Zsoll}(t)}$

These two matrices determine the dynamic model. For the purposes of experimentation, the dynamic model parameters were set in accordance with the table:

Table 1: Parameter set of the system trajectory

Subsystem	Coefficient	Value
$A^{4\times4}$	K_{VR}	0.5
	K_{RB}	0.3
	K_{BZ}	0.45
	K_{ZA}	0.45
$B^{4\times2}$	κ_{VR}	1.1
	κ_{ZA}	0.3
Controller	K_P	1.2
	K_I	0.3
	K_D	0.8

9. TESTING OF THE SIMULATION ENVIRONMENT

Once the corresponding program code has been implemented, initial testing, analysis and adjustment of system parameters can be carried out. This includes, in particular, simulation runs under different configurations of the controller block. P, PI and PID controllers – each implemented with and without low-pass filtering – are systematically tested. The resulting trajectories of the measured variables are then examined and subjected to an initial qualitative assessment. The objective is to identify distinctive dynamic characteristics such as pronounced power peaks, persistent oscillations steady-state deviations, and also the tracking performance.

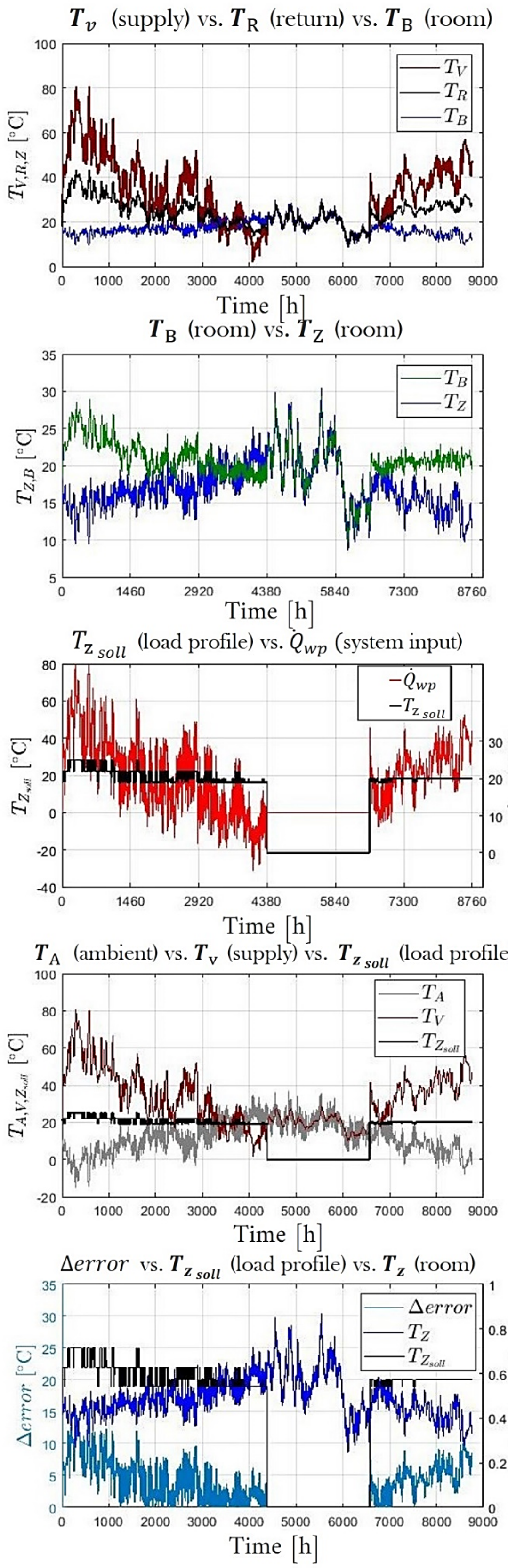

Fig. 9: Measurement when using a P controller

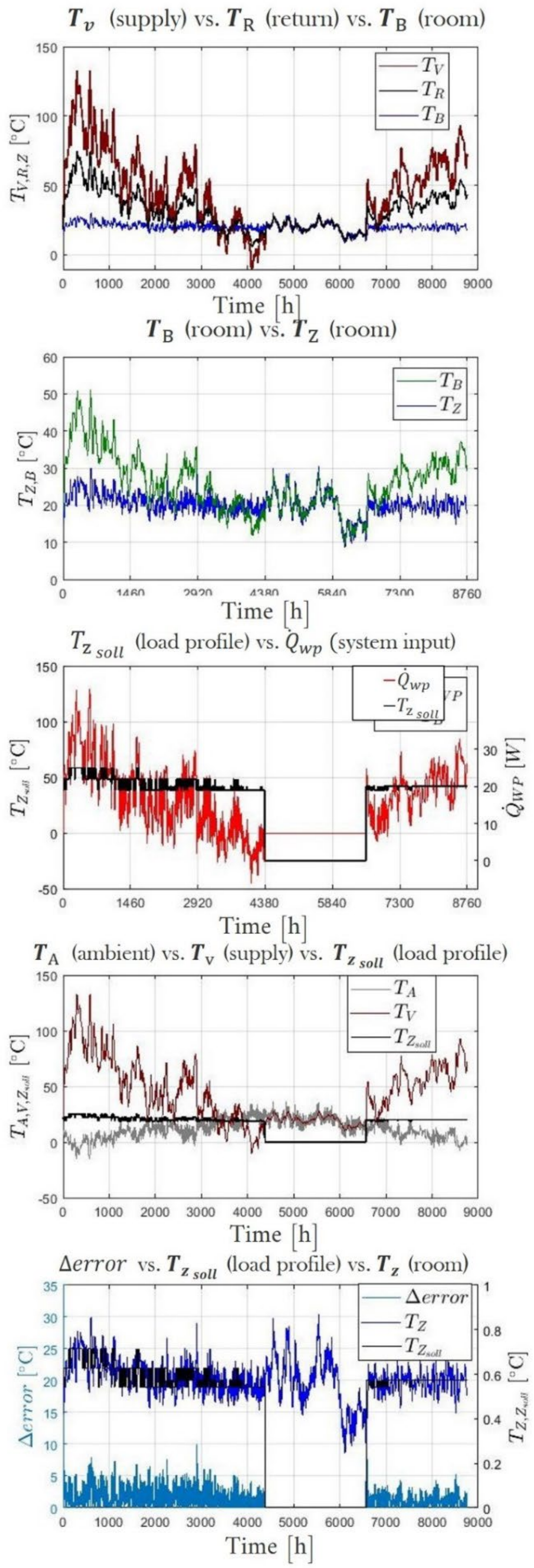

Fig. 10: Measurement when using a PI controller

10. SUMMARY

The primary guiding principle behind the design calculation presented here is the development of a methodology that enables the operational design of heat pumps without resorting to a dedicated dynamic model. Classic thermodynamic models are often very complex and costly to use, while idealised efficiency indicators such as the COP are of limited value. This raises the question of how heat pumps may be reliably evaluated and designed when any detailed model is unavailable. The approach presented here draws on data that is practically always at hand, in the form of manufacturer specifications. Technical manuals typically contain experimentally determined COP curves and operating points. Transferring this data to a structured solution space creates an observational calculation that can be used in time-series simulations. Based on this, load, power consumption and performance can be quantified without the need for a complete thermodynamic model.

The particular appeal of this “detour” is that it enables precise and application-oriented design. Synthetic and real load profiles as well as different environmental conditions can be simulated via mapping to the solution space. This provides access to a comprehensive state overview, allowing comparison of alternative plant designs, testing of control concepts and analysis of operational targets. During the simulation, various controller structures and parametrisations were tested, including proportional (P), proportional-integral (PI) and advanced filtering variants. The simulations show that characteristic system properties - such as power peaks, oscillations or deviations from the target curve - can be clearly identified. The decisive factor here is not so much the exact shape of individual curves as the ability to obtain a consistent status reading from lean data. These reflections have led to a tool enabling targeted design of heat pumps, the testing of control strategies and the evaluation of operating concepts under realistic constraints.

In addition to its practical applicability, the method presented also has a slight aesthetic appeal. The COP observer can distinguish itself through the simplicity of thits underlying idea. In a way, it is also a sleight of hand that alienates a geometrisation technique and uses it to weave sparse data into a practical information carrier in the form of a dynamical estimator. It is certainly practical, useful, and effective, but more important than that is the elegance and simplicity of the design concept itself. Because ultimately, it is not a heat pump but - following Fyodor M. Dostoevsky’s Prince Myshkin - it is the beauty that will save the world.