COMPARABILITY AS A THEORETICAL PRECONDITION FOR SUSTAINABILITY DECISION

VERGLEICHBARKEIT ALS THEORETISCHE VORAUSSETZUNG FÜR NACHHALTIGKEITSENTSCHEIDUNGEN

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SUMMARY

An important class of problems to be dealt with in the course of sustainability considerations consists of questions in which, from a given number of options that can all serve one and the same need-satisfying purpose, the one that appears to be the best possible in the sense of an existing catalogue of criteria is to be selected. The fact that such a selection decision is possible within a scientific framework is bound to certain preconditions. In particular, the mutual comparability of all options considered must be ensured. This paper addresses fundamental aspects of this required comparability.

ZUSAMMENFASSUNG

Eine wichtige Klasse von Problemstellungen, die im Zuge von Nachhaltigkeitsbetrachtungen zu bearbeiten sind, besteht aus Fragestellungen, bei denen aus einer jeweils gegebenen Anzahl von Optionen, die sämtlich ein und demselben bedürfnisbefriedigenden Zweck dienen können, diejenige auszuwählen ist, die im Sinne eines vorliegenden Kriterienkatalogs als die bestmögliche erscheint. Dass eine solche Auswahlentscheidung innerhalb eines wissenschaftlichen Rahmens möglich ist, ist an gewisse Voraussetzungen gebunden. Insbesondere die jeweilige wechselseitige Vergleichbarkeit aller betrachteten Optionen muss sichergestellt sein. Der vorliegende Aufsatz thematisiert grundsätzliche Aspekte dieser geforderten Vergleichbarkeit.

1. INTRODUCTION

It is assumed in the following that several different options are available to fulfil a concrete purpose. In particular, the purpose can be the provision of some product, the use of which provides a concrete function in a specified quantity. This circumstance is also referred to as a "functional unit". In this special case, the different options for action mentioned consist precisely in the respective production of different products with a corresponding functional unit. Thus, a set $\boldsymbol{A}$ of different options for action $\boldsymbol{a_i}$, is considered, which is to be ordered with the help of a suitable real-valued evaluation function $\boldsymbol{F(a_i)}$ in such a way that ultimately the element with the best possible evaluation can be selected. If such a valuation function is available, the decision is made in favour of the option $\boldsymbol{a_n}$ for which the following holds: $\boldsymbol{F(a_n) > F(a_i)}$ for all $\boldsymbol{i \neq n}$ and thus $\boldsymbol{a_n \succ a_i}$, where the operator $\succ$ symbolises the required preference relation and can be read as "is better than".

- “Strong preference" exists if an option $\boldsymbol{a_x}$ is significantly "better" than a competing option $\boldsymbol{a_y}$, so that therefore: $\boldsymbol{a_x \succ a_y}$.
- "Weak preference" exists if an option $\boldsymbol{a_x}$ is "better or as good" as a competing option $\boldsymbol{a_y}$, so that therefore: $\boldsymbol{a_x \succcurlyeq a_y}$.
- "Indifference" exists when a decision cannot be made regarding the options $\boldsymbol{a_x}$ and $\boldsymbol{a_y}$, i.e. $\boldsymbol{a_x \sim a_y}$ applies.

The approach described above is confronted with some problems in the case of a sustainability assessment:

- The evaluation function $\boldsymbol{F(a_i)}$ must be determined in accordance with a heterogeneous catalogue of criteria that is intended to meet the sustainability goals as a whole, whereby on one hand a large number of individual criteria must be taken into account, which on the other hand are not directly quantifiable in all cases.
- A possible indifference with regard to two options $\boldsymbol{a_n \sim a_m}$ cannot be ruled out and, if the case arises, stands in the way of the unambiguousness of the intended decision-making.

- In the case of an evaluation based on a comprehensive catalogue of criteria, the concept of weak preference usually proves to be "too" weak. Weak preference can therefore generally not be used as a basis for decision-making, but rather serves as a justification for further research efforts. Weak preference is therefore not considered in the following.

2. MULTI-CRITERIA EVALUATION

Given is a set $\boldsymbol{B}$ of finitely many criteria $\boldsymbol{b_j}$ with $\boldsymbol{j} \leq \boldsymbol{j_{\max}}$. It is assumed that none of the evaluation criteria $\boldsymbol{b_j}$ falls into the group of so-called "exclusion criteria" and that none of the options for action $\boldsymbol{a_i}$ to be evaluated already fails because of another exclusion criterion. For each $\boldsymbol{a_i}$ a measure (an "indicator") $\boldsymbol{c_j(a_i)}$ can be specified by which the criterion $\boldsymbol{b_j}$ can be quantified or at least evaluated in a comparative manner. Under these conditions one obtains:

$$\boldsymbol{a_x \succ a_y \leftrightarrow c_j(a_x) > c_j(a_y)} \text{ for all } \boldsymbol{j = 1, 2, \ldots, j_{\max}} .$$

It is now obvious that there can be pairs of options for which neither $\boldsymbol{a_x \succ a_y}$ nor $\boldsymbol{a_y \succ a_x}$ is valid, namely in the case of countervailing effects of at least two of the criteria used. Formally speaking, the order based on the preference relation considered here is not a total order, but only a half order or partial order. This circumstance can be illustrated in such a way that, if appropriate, there may be no comparability of the options examined.

To overcome the difficulties described above, weighting factors $\boldsymbol{d_j}$ are now introduced in a further step, by means of which different relevancies of the criteria $\boldsymbol{b_j}$ can be taken into account. Under these circumstances, the evaluation function $\boldsymbol{F(a_i)}$ takes the following form:

$$\boldsymbol{F(a_i, B) = \sum_{j=1}^{j_{\max}} d_j c_j(a_i)} .$$

The chosen notation $\boldsymbol{F(a_i, B)}$ is intended to indicate that the evaluation function $\boldsymbol{F(a_i)}$ considered here is based on the entire catalogue of criteria $\boldsymbol{B}$.

3. MODULES AND SYSTEMS

It was implicitly assumed above that the actions to be evaluated are elementary actions and not complex courses of action that are systematically composed of elementary actions and can be divided into these during the course of an analysis. Elementary actions can also be referred to as "modules" and composite actions as "systems". This designation is explicitly found in the standardisation of the LCA, in that it speaks of process modules and product systems. Elementary actions or processes are represented in the following with symbols of the type $\boldsymbol{a_{i_0}^0}$. The index in superscript indicates that these are actions on the lowest ("$\mathbf{0}$"-th) order. These are numbered with the index $\boldsymbol{i_0}$. Composite actions that occur as parts of even more complex courses of action are accordingly understood as subsystems of more complex systems. Under these circumstances, subsystems of the form $\boldsymbol{a_{i_k}^k}$ must be systematically considered in the evaluation function of a "$\boldsymbol{k}$"-th order system. This can be done in the following way:

$$\boldsymbol{F(a_{i_k}^k, B) = \sum_{j=1}^{j_{\max}} \sum_{i_{k-1}} s_{i_{k-1}} d_j c_j(a_{i_{k-1}}^{k-1})}\,.$$

The summations now take place on the one hand over all criteria and on the other hand over the sub-actions (subsystems) that are part of the overall course of action to be evaluated (overall system). The weighting factors in the equation have different functions. The factors of the form $\boldsymbol{d_j}$ indicate the respective criteria relevance (see above) and are in a way independent of the system under consideration. They are usually called characterisation factors. The factors of the form $\boldsymbol{s_{i_{k-1}}}$, on the other hand, represent the quantification of the inner system coherence. They are called scaling factors. For procedural reasons it is usual to separate the two summation steps. In a first step, the following equation is successively evaluated.

$$\boldsymbol{c_j(a_{i_k}^k) = \sum_{i_{k-1}} c_j(a_{i_{k-1}}^{k-1})}\,.$$

This part of the calculation is referred to as the "life cycle inventory" (LCI) in the context of the LCA. The result of the life cycle inventory is a list of values in the form $\boldsymbol{c_j(a_{i_k}^k)}$, which can now be applied to a valuation function that largely corresponds to the equation given above (see section 2):

$$\boldsymbol{F(a_{i_k}^k, B) = \sum_{j=1}^{j_{\max}} d_j c_j(a_{i_k}^k)}\,.$$

In the context of LCA this step of the calculation is called Life cycle impact assessment (LCIA).

4. INTRANSITIVITY OF INDIFFERENCE

Provided that all values $\boldsymbol{s}_{\boldsymbol{i}_{\boldsymbol{k-1}}}$, $\boldsymbol{d}_{\boldsymbol{j}}$ and $\boldsymbol{c}_{\boldsymbol{j}}\left(\boldsymbol{a}_{\boldsymbol{i}_{\boldsymbol{k}}}^{\boldsymbol{k}}\right)$ can all be specified by single real numbers, it follows that the values of the function $\boldsymbol{F}\left(\boldsymbol{a}_{\boldsymbol{i}_{\boldsymbol{k}}}^{\boldsymbol{k}}, \boldsymbol{B}\right)$ re also single real numbers. Under these circumstances, the options for action $\boldsymbol{a}_{\boldsymbol{i}}$ form a total order with the consequence that the optimum element $\boldsymbol{a}_{\boldsymbol{n}}$ can be uniquely identified, unless two or more of the options considered are completely identical. This means that there is a strict total order of options for action. However, this generally contradicts experience, since it is well known that the phenomenon of insufficient discriminatory precision occurs even in the case of much simpler problems. In the case of multi-criteria evaluations, where comparability can only be achieved by introducing subjective and therefore imprecise weighting factors (see above), this problem is completely unavoidable.

The insufficient discriminatory precision has the consequence that some pairs of options are to be classified as indifferent to each other. In contrast to identity, which practically never exists, indifference is an intransitive relation. It applies:

$$\boldsymbol{a}_{\boldsymbol{x}} \sim \boldsymbol{a}_{\boldsymbol{y}} \wedge \boldsymbol{a}_{\boldsymbol{y}} \sim \boldsymbol{a}_{\boldsymbol{z}} \nrightarrow \boldsymbol{a}_{\boldsymbol{x}} \sim \boldsymbol{a}_{\boldsymbol{z}} \text{ .}$$

If neither a decision between option $\boldsymbol{a}_{\boldsymbol{x}}$ and option $\boldsymbol{a}_{\boldsymbol{y}}$ nor a decision between option $\boldsymbol{a}_{\boldsymbol{y}}$ and option $\boldsymbol{a}_{\boldsymbol{z}}$ möglich ist, is possible, it does not follow that a decision between options $\boldsymbol{a}_{\boldsymbol{x}}$ and $\boldsymbol{a}_{\boldsymbol{z}}$ is simultaneously impossible. This circumstance cannot be properly taken into account with the concepts of ordinary order relations (total order, partial order). The systematic introduction of intervals instead of the usual single real numerical values changes the situation. The application of interval based arithmetic methods in the field of ecological live cycle assessment is dealt with in detail in [1]. In this case, semiorders and interval orders occur, which can serve as a basis for a consistent treatment of the valuation problem addressed here. The applicability of the concept of semiorder in the context of rational choice theory was first explored in a 1956 paper by R.D. Luce [2]. A comprehensive overview of the subject area is given in the monograph [3] by M. Pirlot and Ph. Vincke.

5. INTERVAL BASED ORDER RELATIONS

A closed interval is a continuous set of values extending from a lower limit $\boldsymbol{x_u}$ to an upper limit $\boldsymbol{x_o}$. Formal notation:

$$[\boldsymbol{x_u}; \boldsymbol{x_o}] \coloneqq \{\boldsymbol{x} \in \mathbb{R}^+ | \boldsymbol{x_u} < \boldsymbol{x} < \boldsymbol{x_o}\}\,.$$

The restriction to the range of positive real numbers $\mathbb{R}^+$ is not a relevant restriction at this point, but serves to generally avoid interval arithmetic problems that can occur in connection with intervals that contain the zero point. An order relation ("interval order") suitable for the purposes examined here can be introduced as follows:

$$[\boldsymbol{x_u^a}; \boldsymbol{x_o^a}] > [\boldsymbol{x_u^b}; \boldsymbol{x_o^b}] \leftrightarrow \boldsymbol{x_u^a} > \boldsymbol{x_o^b}\,.$$

An order of this kind is usually "only" a half-order, since overlapping intervals often occur. The descriptive circumstance of overlapping is nothing more than a formal expression of the indifference that may exist:

$$[\boldsymbol{x_u^a}; \boldsymbol{x_o^a}] \sim [\boldsymbol{x_u^b}; \boldsymbol{x_o^b}] \leftrightarrow \boldsymbol{x_o^a} > \boldsymbol{x_u^b} \wedge \boldsymbol{x_o^b} > \boldsymbol{x_u^a}\,.$$

If the intervals to be ordered are required to have the same interval width $\boldsymbol{\Delta} = \boldsymbol{x_o} - \boldsymbol{x_u}$, the interval order changes into a so-called "semiorder" (not to be confused with "half-order" = "partial order"!). $\boldsymbol{\Delta}$ can be understood or introduced here, for example, as the "measurement" or "observation" accuracy. In cases where a constant interval width $\boldsymbol{\Delta}$ does not exist, the theory of semi-orders cannot be applied. In the case of the multi-criteria assessments considered here, this is practically always the case, so that the more comprehensive concept of general interval orders must be brought to bear.

It is further assumed that all values $\boldsymbol{s_{i_k}}$, $\boldsymbol{d_j}$ and $\boldsymbol{c_j(a_{i_k}^k)}$ are not given in the form of individual real numbers, but in the form of intervals:

$$\boldsymbol{s_{i_k}} = [\boldsymbol{s_{i_k u}}; \boldsymbol{s_{i_k o}}]\,,$$

$$\boldsymbol{d_j} = [\boldsymbol{d_{ju}}; \boldsymbol{d_{jo}}]\,,$$

$$\boldsymbol{c_j(a_{i_k}^k)} = [\boldsymbol{c_j(a_{i_k u}^k)}; \boldsymbol{c_j(a_{i_k o}^k)}]\,.$$

The evaluation functions given above can be used without further modifications if the calculation rules of interval arithmetic are observed. The function values are now also intervals:

$$\boldsymbol{F}\left(\boldsymbol{a}_{i_k}^{k}, \boldsymbol{B}\right) = \left[\boldsymbol{F}_{\boldsymbol{u}}\left(\boldsymbol{a}_{i_k}^{k}, \boldsymbol{B}\right); \boldsymbol{F}_{\boldsymbol{o}}\left(\boldsymbol{a}_{i_k}^{k}, \boldsymbol{B}\right)\right].$$

6. EVALUATION

The result intervals that arise according to the presented method for different options $\boldsymbol{a_i}$ can be illustrated along a number line. The simplest case that can occur in this course is shown in Fig. 1.

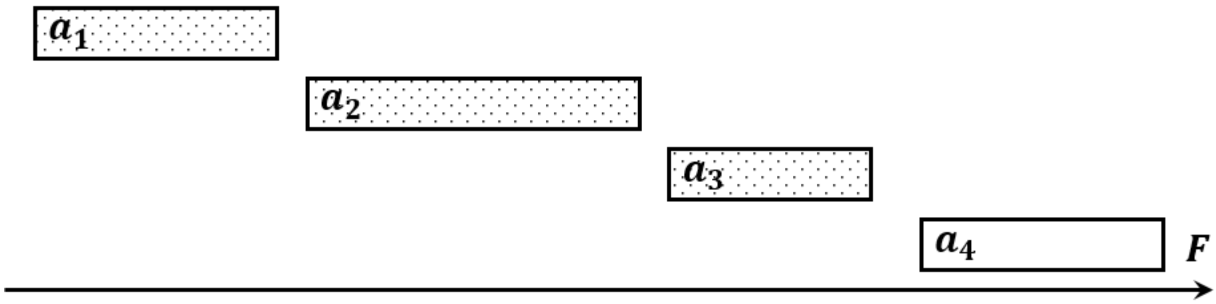

Fig. 1: Strict order

In this case, $\boldsymbol{a_4} \succ \boldsymbol{a_n}$ obviously applies for all $\boldsymbol{n} \neq \boldsymbol{4}$. It is thus clear that the option $\boldsymbol{a_4}$ represents the optimum and should be executed.

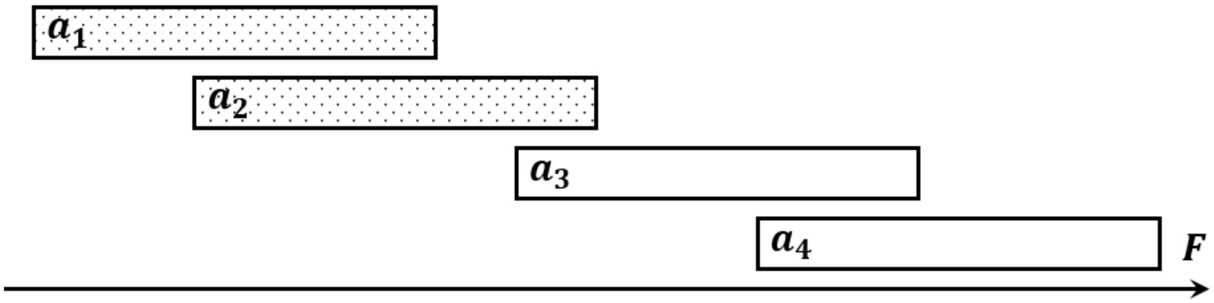

Fig. 2: Semiorder

Fig. 2 shows a situation in which, on the one hand, all intervals have identical widths and in which, on the other hand, the individual intervals partially overlap. A clear decision in favour of option $\boldsymbol{a_4}$ cannot be made in this case. Rather, $\boldsymbol{a_4} \sim \boldsymbol{a_3}$ applies. The two remaining options for action are out of the question because $\boldsymbol{a_4} \succ \boldsymbol{a_1}$ and $\boldsymbol{a_4} \succ \boldsymbol{a_2}$.

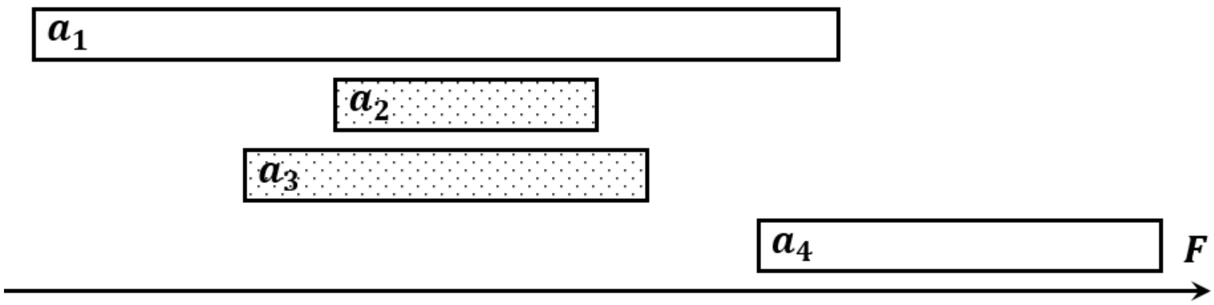

Fig. 3: Interval order

Fig. 3 represents a situation in which the intervals have very different widths and can thus overlap in a much more complex way. In the present case, $\boldsymbol{a_4} \succ \boldsymbol{a_2}$ and $\boldsymbol{a_4} \succ \boldsymbol{a_3}$ as well as $\boldsymbol{a_4} \sim \boldsymbol{a_1}$ apply. It follows that $\boldsymbol{a_1}$ and $\boldsymbol{a_4}$ form the set of optimal options. At first glance, this seems to be in clear contradiction to the fact that option $\boldsymbol{a_1}$ includes the lowest values ever presented. However, according to the theory presented here, this is not sufficient to designate option $\boldsymbol{a_4}$ exclusively as the optimum. Such an evaluation result must rather be interpreted in such a way that the investigation carried out must be refined until a separation of the considered intervals occurs. As a rule, this means that the underlying catalogue of criteria must be critically examined to see whether the options considered are actually mutually comparable in the individual case. If this is not the case, the concrete attempt to identify one of the options for action on the basis of scientific methodology fails. In such cases, one must resort to non-scientific decision-making procedures.

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