STATISTICAL ANALYSIS OF THE SHEAR STRENGTH OF GLUED LAMINATED TIMBER BASED ON FULL-SIZE FLEXURE TESTS

STATISTISCHE AUSWERTUNG VON SCHUBVERSUCHEN AN BRETTSCHICHTHOLZ IN BAUTEILGRÖßE

ANALYSE STATISTIQUE D'ESSAIS DE CISAILLEMENT EN VRAIE GRANDEUR SUR BOIS LAMELLE-COLLE

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SUMMARY

The paper addresses the statistical analysis of the shear strength of structural glued laminated timber (glulam) based on full-size flexure tests. In the study conducted, a total of thirty combined glulam beams (outer three laminations: strength class C35, respectively, six core laminations : strength class C24) were tested. A subset of eighteen specimens failed in the targeted shear mode whereas the remainder failed in flexural mode. In the statistical framework of incomplete data, the random sample obtained was identified as one including Type I right-censored observations. Although such data sets occur under a variety of circumstances in material testing, adequate evaluation procedures are rarely applied. Hence, a major part of the paper is devoted to the appropriate parameter estimation of such random samples. Since statistical distributions of the location-scale family lead to convenient closed-form equations, this case is emphasized. A two-parameter Weibull distribution being of log-location-scale form was found to fit the data adequately.

Statistical inference was based on likelihood ratio procedures. The accuracy of the point estimate for the Weibull distribution was characterized by means of its joint confidence region as well as by means of its marginal confidence intervals. The characteristic shear value of the glulam tested was derived as a lower one-sided confidence interval for the 5%-quantile of the Weibull distribution fitted. The characteristic shear value obtained was found to be slightly above the value given in the new German timber design code DIN 1052.

ZUSAMMENFASSUNG

Der Aufsatz beschäftigt sich mit der statistischen Auswertung von Schubversuchen an Brettschichtholz in Bauteilgröße. In einer Versuchsreihe wurden insgesamt 30 kombinierte Brettschichtholzträger (je drei Lamellen in der Biegezug- bzw. Biegedruckzone : Festigkeitsklasse C35, innere sechs Lamellen : Festigkeitsklasse C24) im Schubversuch geprüft. Dabei war bei 18 Proben Schubversagen und bei 12 Proben Biegeversagen maßgebend. In der Statistik wird ein derartiger Datensatz als „unvollständig" bezeichnet, wobei diejenigen Proben, bei denen Biegeversagen auftritt, als rechts zensierte Daten vom Typ I aufgefaßt werden können. Obwohl unvollständige Datensätze in der Materialprüfung unter einer Vielzahl von Umständen auftreten, werden diese doch nur selten unter Anwendung der hierfür geeigneten statistischen Methoden ausgewertet. Aus diesem Grund wird die Parameterschätzung für unvollständige Datensätze mit Typ I rechts zensierten Beobachtungen ausführlich dargestellt. Für statistische Verteilungen der Lage-Skalen-Familie ergeben sich dabei besonders einfache analytische Ausdrücke, so daß dieser Fall herausgestellt wird. Die Auswertung des vorliegenden Datensatzes erbrachte, daß eine zweiparametrische Weibullverteilung, die der Familie der logarithmierten Lage-Skalen-Verteilungen zugeordnet werden kann, die Häufigkeitsverteilung des Datensatzes sehr gut wiedergibt.

Für Zwecke der statistischen Inferenz wurde die Likelihood-Quotienten-Methode gewählt. Die Genauigkeit der Parameterschätzung für die angepaßte Weibullverteilung wurde sowohl mittels einer Vertrauensregion als auch mit Hilfe von Vertrauensintervallen überprüft. Der charakteristische Wert wurde als einseitiges unteres Vertrauensintervall für die 5%-Quantile der Weibullverteilung hergeleitet. Die Auswertung erbrachte, daß die aus den vorliegenden Daten ermittelte charakteristische Schubfestigkeit geringfügig oberhalb desjenigen Rechenwertes lag, der für Brettschichtholz in der Neuausgabe von DIN 1052 angegeben wird.

RESUME

Cet article traite de l'analyse statistique de résultats d'essais de cisaillement en vraie grandeur effectués sur des poutres en bois lamellé-collé. Dans l'étude considérée, un échantillon de 30 poutres lamellé-collé panaché (3 lamelles extérieures de part et d'autre en C35, lamelles centrales en C24) a été testé. Une rupture en cisaillement a été observée pour 18 spécimens, les 12 restants ayant fait l'objet d'une rupture en flexion. Statistiquement, on considère un tel ensemble de données comme «incomplet », néanmoins, l'ensemble des résultats issus du mode de rupture en flexion peut être considéré comme observation censurée à droite de type I. Bien que cette typologie de données incomplètes soit observée dans une grande variété de circonstances dans les essais de matériaux, l'application de méthode d'analyse statistique adéquate est rare. L'estimation de paramètres de distribution appropriés pour de tels ensembles de données est de ce fait traitée de manière détaillée. La loi de distribution logistique (position, échelle) amenant des analyses particulièrement pratique sera mise en avant. Une distribution de Weibull à deux paramètres, s'apparentant à une forme log-logistique, représente l'ensemble des données de manière très adéquate.

La méthode du maximum de vraisemblance a été choisie pour analyser l'inférence statistique. La précision de l'estimation des paramètres de la distribution de Weibull a été vérifiée aussi bien à l'aide de la région de confiance qu'au moyen des intervalles de confiance marginaux. La résistance caractéristique est alors déterminée comme l'intervalle de confiance unilatéral inférieur du quantile à 5% de la distribution de Weibull. La résistance caractéristique au cisaillement du bois lamellé-collé ainsi obtenue est très légèrement supérieure à celle publiée dans le nouveau code de dimensionnement allemand DIN 1052 :2004.

KEYWORDS: Glued laminated timber, incomplete observations, type I right-censoring mechanism, likelihood ratio procedures, characteristic shear value

1. INTRODUCTION

The shear strength of structural glued laminated timber (glulam) is frequently determined in full-size flexure tests. In a typical test setup, the glulam beams are supported at both ends and loaded by two single forces until ultimate load is reached. The dimensions are usually chosen so that the glulam beams exhibit a low span-to-depth ratio. Although such a test setup is likely to induce shear failure, it is a characteristic feature of such tests that only a certain proportion of the tested specimens fails in the targeted shear mode while the remainder of the specimens fails in flexure. The shear strength of the specimens which fail in shear is easily obtained. Those specimen, however, which fail in flexural mode only yield a shear stress containing the information of what the shear strength must be at least.

In the statistical analysis of such data sets including both, shear strengths and shear stresses as well, the question arises how the shear stresses can be incorporated adequately in the evaluation. A common engineering approach in such a situation is simply not to distinguish between shear strengths and shear stresses and to evaluate all data combined. An attractive alternative to that approach might be to drop the shear stresses and to analyze only the subset of the shear strength values. As the paper will reveal, both engineering approaches are inappropriate. While the first approach overestimates the information content the shear stresses provide by treating them as “strength values”, the second approach takes no advantage of the information the shear stresses can contribute to the analysis. Taking into account that often considerable costs are involved in material, manufacture and testing of glulam beams, the latter approach appears economically highly ineffective.

In the paper presented, the most important observation schemes of data are shortly discussed. It will turn out that observations for which only the lower bound of the failure strength is known (“strength equals at least a certain failure stress”) are referred to as *Type I right-censored*. The statistical analysis of data including Type I right censored observations is illustrated by the results obtained in a study where a total of thirty full-size structural glulam beams were tested.

As it is common in statistics, the parameters of an appropriate distribution will be estimated first. A thorough model assessment is essential before evaluating the data further. The model assessment will be done by graphical and by analytical means, as well. Statistical inference for the estimated parameters will be performed by measuring their accuracy with a joint confidence region and marginal confidence intervals, respectively. In the framework of semi-probabilistic design of timber structures, the derivation of the characteristic shear value is of particular interest. The only meaningful way to establish characteristic strength values adequately is to define them as lower one-sided confidence intervals for a specified quantile of the chosen distribution. How this is accomplished in the presence of Type I right-censored observations concludes the paper.

2. OBSERVATION SCHEMES OF DATA

In practice, *complete* observations are most frequently encountered and their analysis is well described in most elementary textbooks on statistics.

In contrast hereto, *incomplete* observations are often encountered in the study of lifetime data. The most prevalent type of incomplete observations arises when the exact lifetime of a specimen put on test is not observed but is known to exceed a certain time. Such an observation, for which only a lower bound of the lifetime is known, is referred to as *right-censored*. Right-censored lifetimes might arise, for example, if some specimens put on a Duration-of-Load (DOL-) test are still "alive" at the end of the observation period. Two different cases of right-censoring mechanisms need to be distinguished. An observation is termed *Type I right-censored*, if n specimens are put on test and the experiment is terminated after some time before all specimens have failed. A *Type II right-censoring* mechanism is said to apply, when n units are put on test and the experiment is terminated as soon as r of n specimens have failed.

In a different scenario, a specimen put on a DOL-test is inspected for failure after some time. If the specimen has failed before the first inspection only an upper bound of the failure time is known. Such observations are referred to as *left-censored* data.

In some situations, specimens can be inspected only within certain time intervals. For example, it might be impossible to survey the specimens put on a DOL-test continuously. Instead, it might be more convenient to inspect these specimens daily. If failure occurred within such an inspection interval, the observations are termed *interval-censored* data.

Right-censored, left-censored or interval-censored observations represent the most prominent examples of incomplete data although a variety of other observation schemes exists which also lead to the analysis of incomplete data (e.g. truncated data). Lawless [1] gives a complete overview of possible censoring mechanisms.

The well-established procedures for the statistical analysis of censored lifetime data can be easily transferred to the statistical analysis of mechanical stresses. In the light of the glulam data, the shear strengths represent “observed lifetimes” whereas the shear stresses are equivalent to specimens whose observation of lifetime is terminated before the “actual” lifetime was reached. Hence, the shear stresses can be regarded as Type I right-censored observations. The remainder of this paper deals exclusively with this observation scheme.

3. MATERIAL, TEST SETUP AND TEST RESULTS

In the study conducted, a total of thirty full-size flexure tests on glulam were performed with the target to obtain shear failure. The dimensions of the glulam beams were width · depth · length = 140 mm · 456 mm · 3404 mm. The beams consisted of twelve laminations manufactured with spruce (Picea abies), each having a thickness of 38 mm. In order to prevent flexural failure, the lay-up of the laminations was combined : while the three outermost laminations in the tension and compression zone, respectively, consisted of machine graded timber of the strength class C35 according to the European standard EN 338 [2], the six core laminations were made of machine graded timber of the strength class C24. All specimens were manufactured using full-length laminations without any end joints. A melamin urea-formaldehyde adhesive approved for out-door applications was used for face bonding of the laminations.

The test setup was chosen according to a proposal in [3] and is schematically shown in Fig. 1. The glulam beams were simply supported at both ends. The two loads were applied by hydraulic cylinders at a loading rate so as to reach the ultimate load within 300 ± 60 seconds.

The shear strengths and shear stresses obtained in N/mm^2 were, sorted in increasing order, as follows :

3.40*	3.83	3.87*	4.11	4.13	4.20	4.21*	4.23*	4.33*	4.49*
4.50	4.53	4.53*	4.62*	4.77	4.78	4.80*	4.86*	4.92	4.97
5.02	5.02*	5.09*	5.18	5.33	5.44	5.55	5.55	5.68	5.94

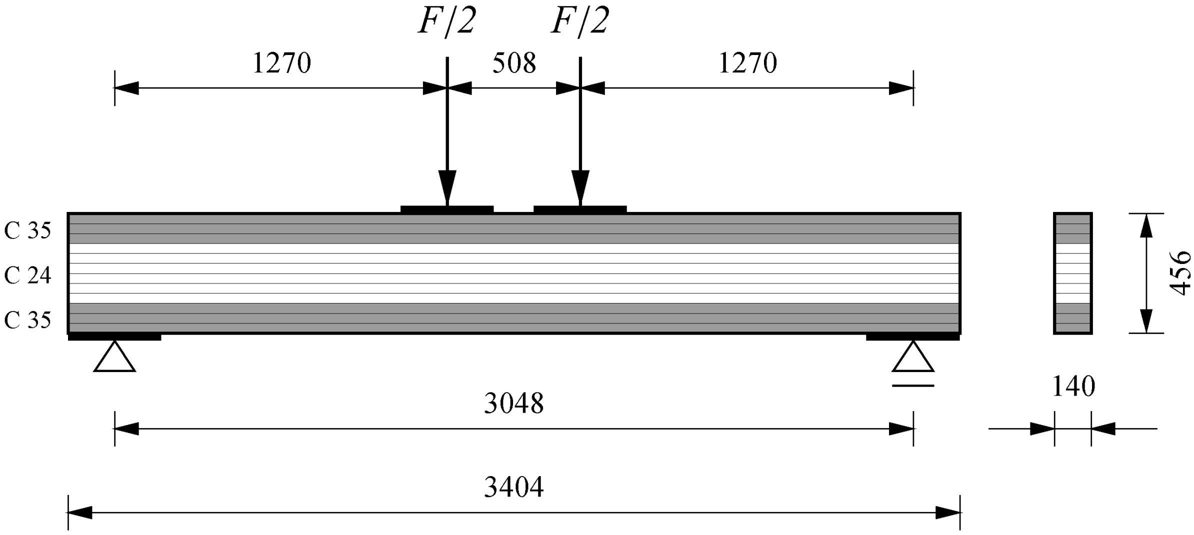

Fig. 1 *Schematic test setup of full-size flexure tests on structural combined glulam*

The asterisks denote Type I right-censored observations (shear stresses at flexural failure). Among thirty specimens tested, only eighteen specimens failed in the targeted shear mode and twelve specimens failed in flexural mode.

4. STATISTICAL ANALYSIS

4.1 Type I Right-Censored Data and Maximum Likelihood Estimates

Suppose, that a random sample X comprises only complete observations which are independent and identically distributed with probability density function (p.d.f.) $f(x;\boldsymbol{\theta})$. The maximum likelihood estimate (m.l.e.) $\hat{\boldsymbol{\theta}}$ is then obtained by maximizing the likelihood function

$$L(\boldsymbol{\theta}) = \prod_{i=1}^{n} f(x_i;\boldsymbol{\theta}) \tag{1a}$$

In almost any case, it is more convenient to work with the log-likelihood function

$$\ell(\boldsymbol{\theta}) = \log L(\boldsymbol{\theta}) \tag{1b}$$

A familiar approach to maximization of the log-likelihood function $\ell(\boldsymbol{\theta})$ is the Newton-Raphson iteration; it uses the iteration scheme

$$\boldsymbol{\theta}_j = \boldsymbol{\theta}_{j-1} - \boldsymbol{H}(\boldsymbol{\theta}_{j-1})^{-1} \cdot \boldsymbol{U}(\boldsymbol{\theta}_{j-1}) \quad , \qquad j = 1,2,.... \tag{2}$$

where $\boldsymbol{U}(\boldsymbol{\theta}) = \partial\ell/\partial\boldsymbol{\theta}$ denotes the first derivative (or score) vector and $\boldsymbol{H}(\boldsymbol{\theta}) = \partial\ell/\partial\boldsymbol{\theta}\,\partial\boldsymbol{\theta}'$ denotes the second derivative (or Hessian) matrix. As a result of the iteration, the m.l.e. $\hat{\boldsymbol{\theta}}$ is obtained. Alternatively, numerical search procedures that do not use any derivatives might be applied to find $\hat{\boldsymbol{\theta}}$.

More generally, suppose that some of the observations in the random sample X are Type I right-censored. Under such a censoring mechanism, the likelihood function takes the form

$$L(\boldsymbol{\theta}) = \prod_{i=1}^{n} f(x_i;\boldsymbol{\theta})^{\delta_i} S(x_i;\boldsymbol{\theta})^{1-\delta_i} \tag{3}$$

where δ_i is called the censoring or status indicator. The censoring indicator is a binary random variable that equals 1 if the observation x_i is uncensored (i.e. shear failure) and that equals 0 if the observation x_i is Type I right-censored (i.e. flexural failure). The term $S(x;\boldsymbol{\theta})$ in eq. (3) denotes the survivor function (s.f.) of the p.d.f. $f(x;\boldsymbol{\theta})$ which is readily obtained by the equation

$$S(x;\boldsymbol{\theta}) = Pr(X \geq x) = \int_{x}^{\infty} f(t;\boldsymbol{\theta})\,dt \tag{4}$$

Thus, for Type I right-censored observations the p.d.f. in the likelihood function $L(\boldsymbol{\theta})$ is merely replaced by its survivor function. It should be noted that under a right-censoring mechanism two random variables are involved : first, the random variable X and second, the binary censoring indicator δ.

4.2 Statistical Model and Parameter Estimation

Over the past decades, the two-parameter Weibull distribution with p.d.f.

$$f(x;\alpha,\beta) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} \exp\left[-(x/\alpha)^{\beta}\right] \quad , \qquad x \geq 0 \tag{5}$$

has gained considerable importance in timber engineering. Hence, this distribution is chosen to model the frequency properties of the data including Type I right-censored observations reported at the end of chapter 3. Instead of working with the random variable X directly, it often turns out to be more convenient to work with the transformed random variable $Y = \log X$. As will become evident, this transformation simplifies the estimation of m.l.e. $\hat{\boldsymbol{\theta}}$ considerably. Performing the transformation $Y = \log X$ (for details see e.g. [4]) yields of what is known as Gumbel (or extreme value) distribution with p.d.f.

$$f(y;u,b) = \frac{1}{b} e^{(y-u)/b} \exp\left[-e^{(y-u)/b}\right] \quad , \qquad -\infty < y < \infty \tag{6}$$

where $u = \log \alpha$ is the location parameter and $b = 1/\beta$ is the scale parameter. The Gumbel distribution has the advantage to be of location-scale form whereas the Weibull distribution is of log-location-scale form. Distributions of the location-scale family (e.g. normal distribution, logistic distribution) have the appealing feature that under a Type I right censoring mechanism simple closed-form equations of the score vector $\boldsymbol{U}(\boldsymbol{\theta})$ and the Hessian matrix $\boldsymbol{H}(\boldsymbol{\theta})$ exist.

For location-scale distributions, the likelihood function eq. (3) including Type I right-censored observations takes the form

$$L(u,b) = \prod_{i=1}^{n} \left[\frac{1}{b} f_0(z_i)\right]^{\delta_i} S_0(z_i)^{1-\delta_i} \tag{7a}$$

where $z_i = (y_i - u)/b = (\log x_i - u)/b$. The functions $f_0(z)$ and $S_0(z)$ in eq. (7a) denote the standardized probability density and survivor function, respectively. The corresponding log-likelihood function $\ell(\boldsymbol{\theta}) = \log L(\boldsymbol{\theta})$ (eq. (1b)) is readily seen to be

$$\ell(u,b) = -r \log b + \sum_{i=1}^{n} \left[\delta_i \log f_0(z_i) + (1-\delta_i) \log S_0(z_i)\right] \tag{7b}$$

where $r = \sum_{i=1}^{n} \delta_i$.

The components of the score vector $\boldsymbol{U}(\boldsymbol{\theta}) = \partial\ell/\partial\boldsymbol{\theta}$ are found to be

$$U_1 = \frac{\partial \ell}{\partial u} = -\frac{1}{b} \sum_{i=1}^{n} \left[\delta_i \frac{\partial \log f_0(z_i)}{\partial z_i} + (1-\delta_i) \frac{\partial \log S_0(z_i)}{\partial z_i}\right] \tag{8a}$$

$$U_2 = \frac{\partial \ell}{\partial b} = -\frac{r}{b} - \frac{1}{b} \sum_{i=1}^{n} \left[\delta_i z_i \frac{\partial \log f_0(z_i)}{\partial z_i} + (1-\delta_i) z_i \frac{\partial \log S_0(z_i)}{\partial z_i}\right] \tag{8b}$$

while the components of the Hessian matrix $\boldsymbol{H}(\boldsymbol{\theta}) = \partial\ell/\partial\boldsymbol{\theta}\,\partial\boldsymbol{\theta}'$ take the form

$$H_{11} = \frac{\partial^2 \ell}{\partial u^2} = \frac{1}{b^2} \sum_{i=1}^{n} \left[\delta_i \frac{\partial^2 \log f_0(z_i)}{\partial z_i^2} + (1-\delta_i) \frac{\partial^2 \log S_0(z_i)}{\partial z_i^2}\right] \tag{9a}$$

$$H_{22} = \frac{\partial^2 \ell}{\partial b^2} = \frac{r}{b^2} + \frac{2}{b^2} \sum_{i=1}^{n} \left[\delta_i \, z_i \, \frac{\partial \log f_0(z_i)}{\partial z_i} + (1 - \delta_i) \, z_i \, \frac{\partial \log S_0(z_i)}{\partial z_i} \right]$$

$$+ \frac{1}{b^2} \sum_{i=1}^{n} \left[\delta_i \, z_i^2 \, \frac{\partial^2 \log f_0(z_i)}{\partial z_i^2} + (1 - \delta_i) \, z_i^2 \, \frac{\partial^2 \log S_0(z_i)}{\partial z_i^2} \right] \quad (9b)$$

$$H_{12} = \frac{\partial^2 \ell}{\partial u \, \partial b} = \frac{1}{b^2} \sum_{i=1}^{n} \left[\delta_i \, \frac{\partial \log f_0(z_i)}{\partial z_i} + (1 - \delta_i) \frac{\partial \log S_0(z_i)}{\partial z_i} \right]$$

$$+ \frac{1}{b^2} \sum_{i=1}^{n} \left[\delta_i \, z_i \, \frac{\partial^2 \log f_0(z_i)}{\partial z_i^2} + (1 - \delta_i) \, z_i \, \frac{\partial^2 \log S_0(z_i)}{\partial z_i^2} \right] \quad (9c)$$

$$H_{21} = H_{12} \, . \qquad (9d)$$

The standardized probability density and survivor function of the Gumbel distribution can be expressed as

$$f_0(z) = e^z \exp\left(-e^z\right), \qquad S_0(z) = \exp\left(-e^z\right) \qquad (10a,b)$$

The first and second derivatives of $\log f_0(z)$ and $\log S_0(z)$ needed for calculating the components of the score vector $\boldsymbol{U}(\boldsymbol{\theta})$ (eqs. (8a,b)) and the Hessian matrix $\boldsymbol{H}(\boldsymbol{\theta})$ (eqs. (9a-d)) follow from eqs. (10a,b) immediately

$$\frac{\partial \log f_0(z)}{\partial z} = 1 - e^z, \qquad \frac{\partial^2 \log f_0(z)}{\partial z^2} = -e^z \qquad (11a,b)$$

$$\frac{\partial \log S_0(z)}{\partial z} = -e^z, \qquad \frac{\partial^2 \log S_0(z)}{\partial z^2} = -e^z \qquad (11c,d)$$

With the components of the score vector $\boldsymbol{U}(\boldsymbol{\theta})$ and the Hessian matrix $\boldsymbol{H}(\boldsymbol{\theta})$ known, the Newton-Raphson iteration according to eq. (2) can be performed. For the data under consideration, the m.l.e. $\hat{\boldsymbol{\theta}} = \left(\hat{u}, \hat{b}\right) = (1.666, 0.0919)$ of the Gumbel distribution is obtained.

The simple transformations $\hat{\alpha} = \exp(\hat{u}) = \exp(1.666) = 5.289$ and $\hat{\beta} = 1/\hat{b} = 1/0.0919 = 10.876$ finally yield the m.l.e. of the Weibull distribution.

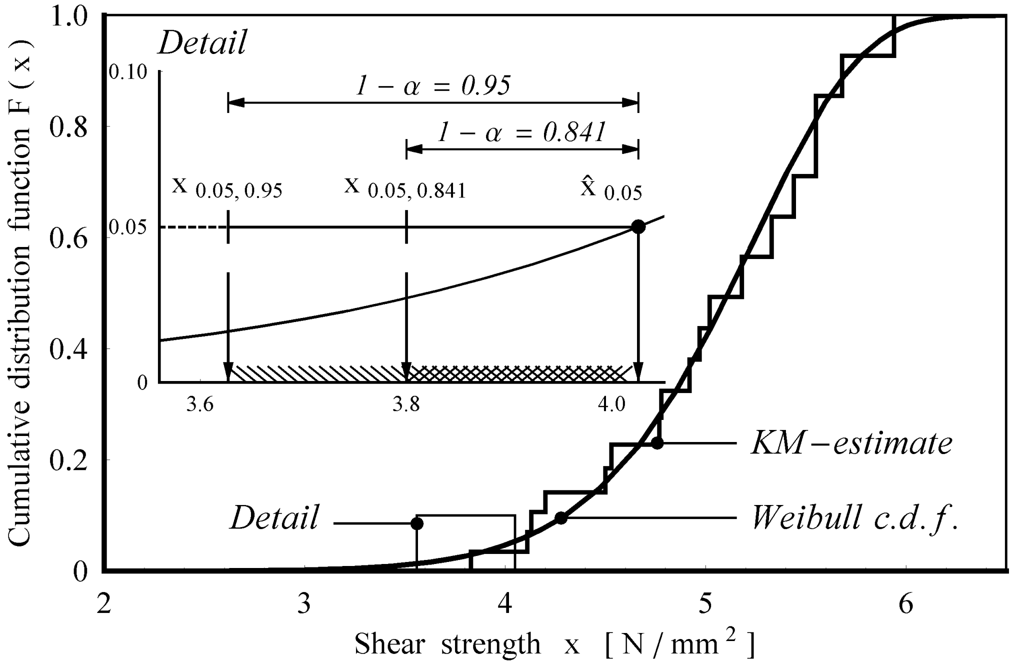

Fig. 2 *Nonparametric Kaplan-Meier (KM-) estimate and Weibull cumulative distribution function. Approximate one-sided lower confidence intervals for the 0.05-quantile at confidence levels $1 - \alpha = 0.841$ and $1 - \alpha = 0.95$ are shown as detail.*

4.3 Model Assessment

Before performing statistical inference, a thorough assessment of the parametric model is essential. Descriptive plots are a common tool for model assessment. Although being subjective, they provide a useful impression of the appropriateness of the chosen parametric model. Usually, a formal goodness-of-fit test supplements such plots. Here, the latter is omitted in favour of an analysis of the shape parameter of the generalized log-gamma distribution which allows a discrimination between the Weibull distribution on the one hand and the log-normal distribution on the other hand. The latter distribution is also frequently applied in timber engineering.

4.3.1 Graphical Model Assessment

Nonparametric frequency estimates for complete random samples are well-known. In case of a random sample including Type I right-censored observations, however, these estimates are not applicable. Kaplan and Meier proposed in 1958 an approach which allows a nonparametric estimate of the cumulative distribution function (c.d.f.) for any right-censored random sample. For details of the calculation, see [1,5]. In Fig. 2, the Kaplan-Meier estimate is plotted as a step function with the Weibull c.d.f. overlaid. As can be seen, there is no graphical evidence against the two-parameter Weibull model.

4.3.2 Analytical Model Assessment

The generalized log-gamma distribution is a further representative of the location-scale distribution family with location parameter u and scale parameter b. In addition, it includes a shape parameter k. The shape parameter has the interesting feature to allow a discrimination between the Weibull distribution and the log-normal distribution. Letting $Y = \log X$ and $Z = (Y - u)/b$, the p.d.f. and s.f. of the generalized log-gamma distribution are given in standardized form as

$$f_0(z;k) = \frac{k^{k-1/2}}{\Gamma(k)} \exp\left[k^{1/2}\, z - k \exp\left(z\, k^{-1/2}\right)\right] \tag{12a}$$

$$S_0(z;k) = 1 - I\left(k,\, k \exp\left(z\, k^{-1/2}\right)\right) \tag{12b}$$

where $\Gamma(k) = \int_0^\infty u^{k-1} e^{-u}\, du$ and $I(k,x) = \frac{1}{\Gamma(k)} \int_0^x u^{k-1}\, e^{-u} du$ denote the complete and incomplete gamma function, respectively.

In the limit as the shape parameter $k \to \infty$, $f_0(z;k)$ approaches the p.d.f. of the standard normal distribution and as $k \to 1$, $f_0(z;k)$ approaches the p.d.f. of the Gumbel distribution. Similarly, the same holds true for the survivor function. Recall, that the normal and the log-normal distribution as well as the Gumbel and the Weibull distribution are related to each other by the simple transformation rule $Y = \log X$, respectively. Therefore, the shape parameter k allows a discrimination beetween the log-normal and Weibull distribution, too.

Performing the Newton-Raphson iteration according eq. (2) in association with the score vector and Hessian matrix for location-scale models presented in section 4.2, the maximum likelihood estimate of the generalized log-gamma distribution is found to be $\hat{\boldsymbol{\theta}} = (u, b, k) = (1.664, 0.0929, 1.069)$. The shape parameter $k = 1.069$ is very close to 1 which indicates that the two-parameter Weibull model is appropriate. In contrast hereto, the log-normal model would be inappropriate for the Type I right-censored random sample under consideration.

4.4 Statistical Inference

Exact statistical inference procedures for random samples including Type I right-censored observations are mathematically intractable. There exists, however, a variety of approximate methods for the analysis of such data. Among the most important are score procedures, maximum likelihood based procedures and likelihood ratio procedures. Wald-based inference procedures assuming approximately normally distributed pivotal quantities as well as bootstrap methods which apply simulation of the distributional properties of pivotal quantities provide convenient alternatives.

Subsequently, the likelihood ratio statistic defined by

$$\Lambda(\boldsymbol{\theta}) = -2\log\left[\frac{L(\boldsymbol{\theta})}{L(\hat{\boldsymbol{\theta}})}\right] = 2\,\ell\left(\hat{\boldsymbol{\theta}}\right) - 2\,\ell\left(\boldsymbol{\theta}\right) \tag{13}$$

is favoured. In large samples, the maximum likelihood estimate $\hat{\boldsymbol{\theta}}$ is approximately multivariate normally distributed with $N_p\left(\boldsymbol{\theta};\, I^{-1}(\boldsymbol{\theta})\right)$ where $I(\boldsymbol{\theta}) = E\left(-\partial^2\ell / \partial\boldsymbol{\theta}\,\partial\boldsymbol{\theta}'\right)$ denotes the Fisher (or expected) information matrix. The likelihood ratio statistic $\Lambda(\boldsymbol{\theta})$ is then approximately χ^2-distributed.

4.4.1 Approximate joint confidence region for $\hat{\boldsymbol{\theta}}$

The m.l.e. of a parameter vector is of little value unless it is known how accurate it is likely to be. Hence, measurement of the extent of this uncertainty is an important part of the statistical problem. First, let us obtain an approximate joint confidence region for the m.l.e. $\hat{\boldsymbol{\theta}} = \left(\hat{u}, \hat{b}\right) = (1.666, 0.0919)$ of the Gumbel distribution discussed in section 4.2. In terms of likelihood ratio procedures, an approximate joint confidence region is given as the contour of all points (u, b) satisfying $\Lambda(u, b) \le \chi^2_{p;1-\alpha}$, where

$$\Lambda(u, b) = 2\,\ell\left(\hat{u}, \hat{b}\right) - 2\,\ell(u, b) \tag{14}$$

denotes the likelihood ratio statistic and $\chi^2_{p;1-\alpha}$ is the quantile of the χ^2-distribution with p degrees of freedom and confidence level $1-\alpha$. Figure 3 shows the contour of the approximate confidence region for the m.l.e. $\hat{\boldsymbol{\theta}}$ assuming a confidence level of $1-\alpha = 0.95$. As two parameters are involved, the χ^2-distribution has $p = 2$ degrees of freedom. Under these assumptions, the quantile becomes $\chi^2_{2;0.95} = 5.991$.

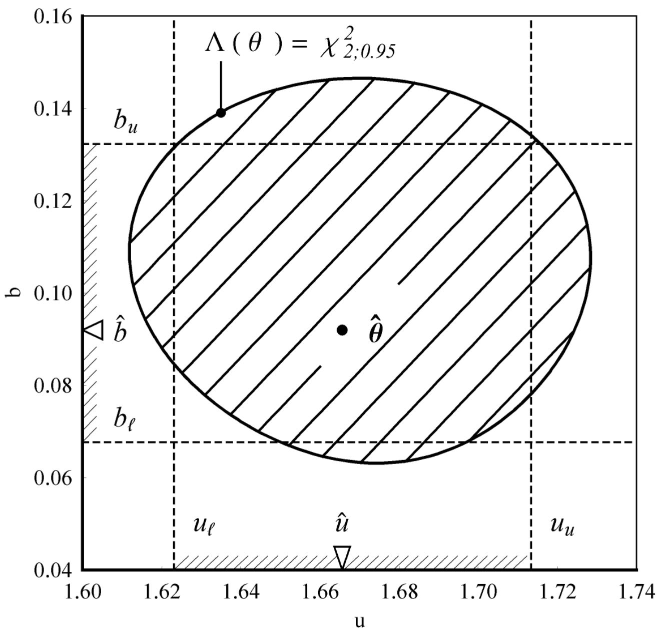

Fig. 3 *Approximate 0.95-joint confidence region (hatched contour) for the maximum likelihood estimate $\hat{\boldsymbol{\theta}}$ and approximate two-sided marginal 0.95-confidence intervals for the parameters $\hat{u}$ and $\hat{b}$ (dashed lines)*

4.4.2 Approximate two-sided marginal confidence intervals for $\hat{\theta}$

An approximate two-sided marginal confidence interval concerning u is obtained by the likelihood ratio statistic

$$\Lambda_1(u_0) = 2\,\ell\left(\hat{u},\hat{b}\right) - 2\,\ell\left(u_0,\tilde{b}\left(u_0\right)\right) \tag{15a}$$

where $\tilde{b}(u_0)$ is the maximum likelihood estimate for b when $u = u_0$. This is obtained by maximizing $\ell(u_0, b)$ with respect to b. The marginal confidence interval concerning u is then given as the set of points satisfying $\Lambda_1(u_0) \le \chi^2_{p;1-\alpha}$. In a similar way, an approximate two-sided marginal confidence interval concerning b is obtained by using

$$\Lambda_2(b_0) = 2\,\ell\left(\hat{u},\hat{b}\right) - 2\,\ell\left(\tilde{u}\left(b_0\right), b_0\right) \tag{15b}$$

where $\tilde{u}(b_0)$ maximizes $\ell(u, b_0)$ when $b = b_0$. Similarly, the marginal confidence interval is given as the set of points satisfying $\Lambda_2(b_0) \le \chi^2_{p;1-\alpha}$. In Figs. 4a,b, the likelihood ratio statistics $\Lambda_1(u_0)$ and $\Lambda_2(b_0)$ as well as their intersections with the limiting quantile $\chi^2_{p;1-\alpha}$ are shown graphically.

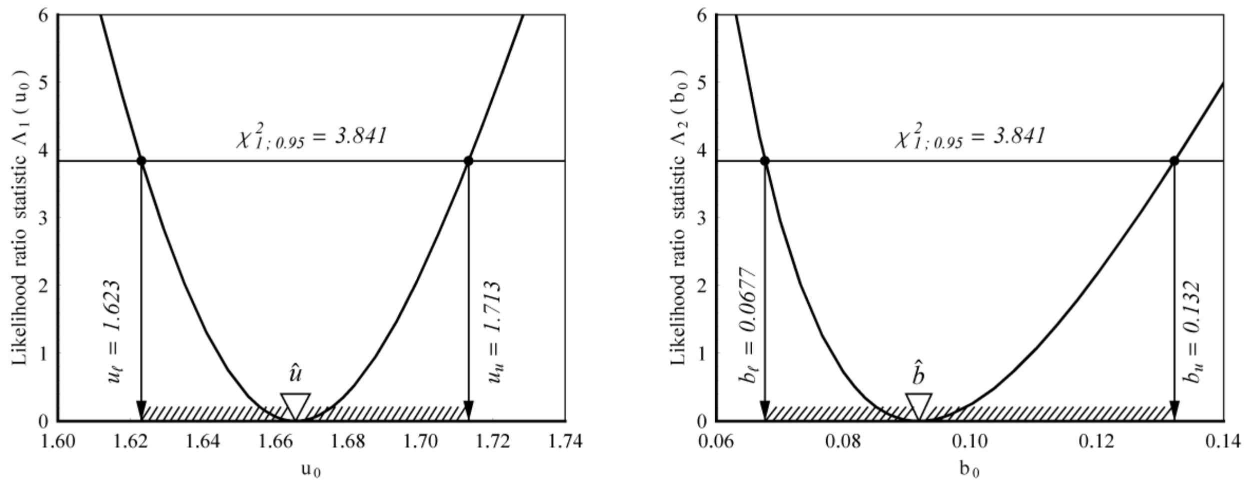

Fig. 4a,b *Approximate two-sided marginal 0.95-confidence intervals for the parameters $\hat{u}$ and $\hat{b}$ obtained by means of the likelihood ratio statistics $\Lambda_1(u_0)$ and $\Lambda_2(b_0)$*

Again, the confidence level was assumed to be $1-\alpha = 0.95$. As the parameters are now considered separately, the degree of freedom of the χ^2-distribution reduces to $p = 1$ so that the quantile becomes $\chi^2_{1;0.95} = 3.841$.

The approximate two-sided marginal 0.95-confidence intervals for u and b are found to be $1.623 \le u \le 1.713$ and $0.0677 \le b \le 0.132$. These marginal confidence intervals are shown in Fig. 3 with lines in dashed style along with the joint confidence region for the m.l.e. $\hat{\boldsymbol{\theta}}$.

The transformations $\alpha = \exp(u)$ and $\beta = 1/b$ yield the approximate two-sided marginal 0.95-confidence intervals for the parameters of the Weibull distribution; they are obtained as $5.069 \le \alpha \le 5.548$ and $7.561 \le \beta \le 14.780$. It can be seen that the confidence interval of the location parameter α is quite narrow indicating its precise estimation. The confidence interval for the scale parameter β, however, is fairly wide thus emphasizing the need of a greater sample size in order to obtain a more precise estimate.

4.4.2 Approximate one-sided lower confidence interval for the 5%-quantile

For location-scale models, the qth quantile $\hat{y}_q$ for $Y = \log X$ is

$$\hat{y}_q = \hat{u} + w_q \cdot \hat{b} \tag{16a}$$

where $\hat{u}$ and $\hat{b}$ are the m.l.e. and $w_q = F_0^{-1}(q)$ denotes the qth quantile of the standardized c.d.f. $F_0(z) = 1 - S_0(z)$. For the Gumbel distribution, the qth quantile of the standardized c.d.f. takes the form

$$w_q = \log\left[-\log(1-q)\right] \tag{16b}$$

The related qth quantile of the Weibull distribution is easily obtained by the transformation $\hat{x}_q = \exp(\hat{y}_q)$.

Considering mechanical strength properties, the 0.05-quantile is usually of particular interest. Inserting the m.l.e. obtained in section 4.2 and eq.(16b) into eq. (16a) yields the quantile for Y (Gumbel distribution)

$$\hat{y}_{0.05} = \hat{u} + \log\left[-\log(1-q)\right]\cdot\hat{b} = 1.666 + (-2.970)\cdot 0.0919 = 1.393 \tag{17a}$$

The 0.05-quantile for X (Weibull distribution) is

$$\hat{x}_{0.05} = \exp(\hat{y}_p) = \exp(1.393) = 4.03 \text{ N/mm}^2 \tag{17b}$$

For the calculation of the quantile according to eq. (16a) the m.l.e. $\hat{\boldsymbol{\theta}} = (\hat{u}, \hat{b})$ is needed. The joint confidence region shown in Fig. 3 illustrates graphically where the m.l.e. $\hat{\boldsymbol{\theta}}$ can be expected to lie in the parameter plane in a repetition of the experiment. From the hatched contour plotted it can be easily seen that the 0.05-quantile derived so far is totally inappropriate to establish a characteristic shear value as it is obtained entirely random. The link between the observed sample quantile and the population is provided by a one-sided lower confidence interval.

An approximate one-sided lower confidence interval concerning $\hat{y}_q$ is obtained by the likelihood ratio statistic

$$\Lambda(y_{q0}) = 2\,\ell(\hat{u},\hat{b}) - 2\,\ell(\tilde{u}(y_{q0}),\tilde{b}) \tag{18a}$$

where $(\tilde{u}(y_{q0}),\tilde{b})$ maximizes $\ell(u,b)$ when $y_q = y_{q0}$. Since for location-scale models the relation $y_q = u + w_q \cdot b$ holds, we merely need to maximize

$$\ell_1(b) = \ell(y_{q0} - w_q \cdot b, b) \tag{18b}$$

with respect to b in order to get $\tilde{b}$, and then $\tilde{u} = y_{q0} - w_q \cdot \tilde{b}$.

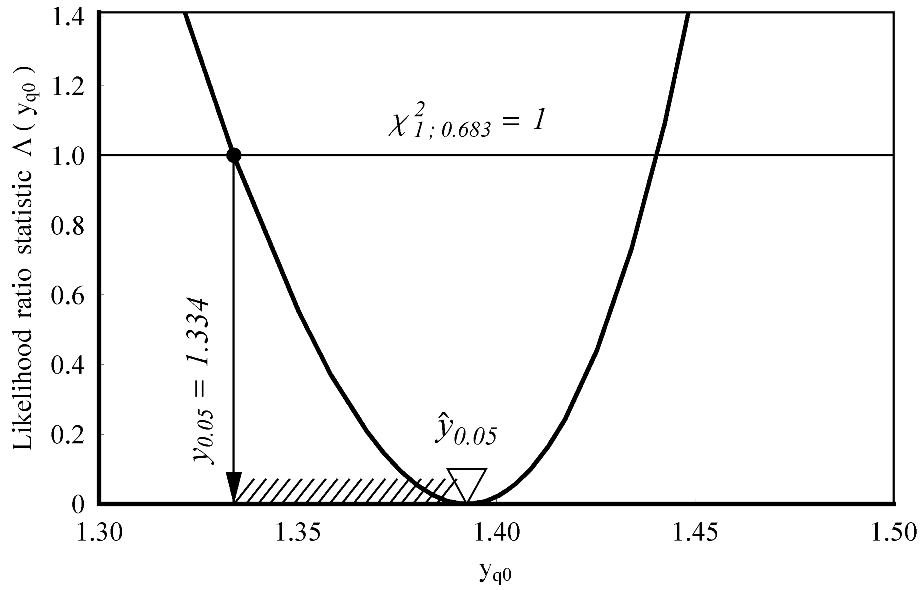

Fig. 5 *Approximate one-sided lower 0.841-confidence interval for the 0.05-quantile obtained by means of the likelihood ratio statistic* $\Lambda\left(y_{q0}\right)$

The approximate one-sided lower confidence interval consists of all values y_{q0} satisfying

$$I\left(y_{q0} < \hat{y}_{q0}\right)\,\Lambda\left(y_{q0}\right) \leq \chi^2_{1;\,1-2\alpha} \qquad (19)$$

where $I\left(y_{q0} < \hat{y}_{q0}\right)$ is a binary indicator function that equals 1 if the unequality is true and 0 if it is not true.

In Fig. 5, the likelihood ratio statistic $\Lambda\left(y_{q0}\right)$ according to eqs. (18a,b) is shown graphically for $q = 0.05$. In many European standards (e.g. EC 5, EN 1058, EN 14358), the confidence level $1-\alpha = 0.841$ is proposed. Under this assumption, the quantile of the χ^2-distribution becomes $\chi^2_{1;\,0.683} = 1$ which is shown as horizontal line in Fig. 5. The left intersection of this line with the likelihood ratio statistic $\Lambda\left(y_{q0}\right)$ yields the lower limit of the one-sided confidence interval.

For the data under consideration, the approximate one-sided lower 0.841-confidence interval is found to be $y_{0.05;0.841} = 1.334$. The transformation to the Weibull distribution yields $x_{0.05;0.841} = \exp\left(1.334\right) = 3.80$ N/mm^2. Alternatively, for the more conservative confidence level $1-\alpha = 0.95$ the lower limit of the one-sided confidence interval becomes $x_{0.05;0.95} = 3.63$ N/mm^2. In Fig. 2, these confidence intervals are shown as detail in the inserted graphics.

5. CONCLUSIONS

In the paper presented, it was reported on a total of thirty full-size flexure tests conducted on structural combined glued laminated timber in order to determine its shear strength. Only 60% of the specimens were found to fail in the targeted shear mode whereas 40% of the specimens failed in flexural mode. The data set obtained, consisting of shear strengths and shear stresses at flexural failure as well, was identified as a random sample including Type I right censored observations.

Random samples including Type I right censored observations are a special case of the more general statistical theory of incomplete data. While statistical evaluation routines for incomplete data are an important topic in many disciplines such as medicine, social sciences as well as in mechanical and electrical engineering, they are rarely if ever applied in timber engineering. Hence, the paper laid particular emphasis on the adequate statistical analysis of the incomplete shear data obtained in the flexural tests conducted on combined glued laminated timber.

Both, graphical and analytical model assessment proved that a two-parameter Weibull distribution, being of log-location-scale form, fitted the data adequately. The parameter estimation and statistical inference, however, were for sake of simplicity performed applying the Gumbel distribution being of location-scale form. For this family of distributions, convenient closed-form equations exist. The results for the Weibull distribution were obtained by means of simple transformation rules, respectively.

The contour of the joint parameter region was plotted in order to illustrate where the estimated parameter vector might be located in a repetition of the experiment. The marginal confidence intervals revealed that the location parameter of the Weibull distribution was estimated rather precisely; the marginal confidence interval for the scale parameter, however, was comparatively wide thus emphasizing the necessity of a greater sample size.

The characteristic shear value was derived as one-sided lower confidence interval for the 5%-quantile of the Weibull distribution. For both confidence levels considered (84.1% and 95%), the characteristic shear values were found to be $f_{\mathrm{v},k} = 3.80\ \mathrm{N/mm}^2$ and $f_{\mathrm{v},k} = 3.63\ \mathrm{N/mm}^2$, respectively. Thus, both values were slightly above the characteristic value $f_{v,k} = 3.5$ N/mm^2 specified in the new German timber design code DIN 1052 [6].

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