DETERMINATION OF CHARACTERISTIC VALUES FOR STRUCTURAL TIMBER

BESTIMMUNG CHARAKTERISTISCHER RECHENWERTE VON BAUHOLZ FÜR TRAGENDE ZWECKE

DÉTERMINATION DES VALEURS CARACTÉRISTIQUES DE DIMENSIONNEMENT DU BOIS DE STRUCTURE

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SUMMARY

This paper addresses the determination of characteristic values for structural timber as stated in the European standard EN 14358. For the user of this standard, however, the specific statistical result of the calculation is not evident, as the terminology applied is misleading. In addition, the fundamentals of the subject are rarely presented in literature. Hence, a comprehensive derivation of the statistical factors underlying the evaluation procedure proposed in EN 14358 is given. Statistical assumptions and the mathematical background are illustrated in full detail. Characteristic values are shown to be lower one-sided confidence intervals for the 5%-quantile; not the 5%-quantile as stated by the current standard.

The tables which present the statistical factors in EN 14358 are rather incomplete and specify numerical values only for a few sample sizes. In order to make the application of EN 14358 more convenient and numerically more precise, complete tables with statistical factors ranging up to a sample size of 100 are given for both confidence levels applied in the standard.

ZUSAMMENFASSUNG

Der Aufsatz beschäftigt sich mit der Bestimmung charakteristischer Rechenwerte von Bauholz für tragende Zwecke. Dabei wird auf das Auswerteverfahren Bezug genommen, welches in der europäischen Norm EN 14358 angegeben ist. Für den Anwender der Norm geht aus dem Normentext jedoch nicht zweifelsfrei hervor, was das Ergebnis der Rechnung im statistischen Sinne ist, da die Terminologie der Norm nicht korrekt ist. Da zudem die Grundlagen des

Auswerteverfahrens in der Fachliteratur, wenn überhaupt, nur bruchstückhaft behandelt werden, wird hier eine vollständige Herleitung der statistischen Faktoren, die dem Verfahren zugrunde liegen, angegeben. Dabei werden sowohl die statistischen Annahmen als auch der mathematische Hintergrund dargestellt. Aus der Herleitung der statistischen Faktoren geht hervor, daß nicht, wie in der Norm angegeben, die 5%-Fraktile, sondern vielmehr ein einseitiges unteres Vertrauensintervall der 5%-Fraktile als charakteristischer Rechenwert erhalten wird.

Die in EN 14358 angegebenen Tabellen sind lückenhaft und weisen nur für einige wenige Stichprobenumfänge statistische Faktoren aus. Um die Anwendung der Norm zu erleichtern, werden vollständige Tabellen bis hin zu einem Stichprobenumfang von 100 Proben für die beiden in EN 14358 angewandten Vertrauensniveaus angegeben.

RÉSUMÉ

Cet article porte sur la détermination des valeurs caractéristiques de dimensionnement pour le bois de structure, telles qu’elles sont définies dans la norme européenne EN 14358. Pour l’utilisateur de cette norme cependant, il n’est pas évident d’appréhender le résultat de ce calcul, au sens statistique, dans la mesure où la terminologie appliquée n’est pas explicite. De plus, les bases théoriques du sujet sont rarement traitées dans la littérature. Cet article propose donc de préciser, de manière compréhensible, les paramètres statistiques qui sous-tendent la procédure d’évaluation de la norme EN 14358. Ainsi, les hypothèses statistiques et les fondements mathématiques sont illustrés de manière détaillée. L’analyse montre clairement que ce n’est pas le fractile à 5%, mais une borne inférieure de l’intervalle de confiance de ce fractile, que l’on obtient comme valeur caractéristique.

Les tableaux présentant les paramètres statistiques dans la norme EN 14358 sont plutôt incomplets, et ne spécifient des valeurs numériques que pour quelques tailles d’échantillonnage. Afin de rendre l’application de la norme EN 14358 plus simple et plus précise, des tableaux complets comportant des paramètres statistiques jusqu’à une taille d’échantillons de 100 sont proposés, pour les deux niveaux de confiance appliqués dans la norme.

KEYWORDS: structural timber, characteristic values, transformation of random variables, noncentral *t*-distribution, quantiles, tolerance intervals

1. INTRODUCTION

In semi-probabilistic design of structures, the proper determination of characteristic values plays a crucial role. For structural timber, the European standard EN 14358 [5] describes a proper evaluation procedure for the determination of characteristic values. However, the terminology applied in the standard is ambiguous as the term "5%-quantile" is used in a misleading manner. Although a footnote provides some explanation, it remains difficult for the user to understand the assumptions on which the statistical factors presented are based, as well as their final effect.

Of course, it is imperative to understand the meaning of such statistical factors before applying them in an evaluation. Hence, the first aim of this paper is to present a comprehensive derivation of the statistical factors as given in EN 14358. Since the tables of this standard provide only a few numerical values, the second aim is to make complete tables available for sample sizes ranging up to 100 specimens and for the two confidence levels proposed in the standard.

2. GENERAL STATISTICAL BACKGROUND

Suppose that X is a normally distributed random variable. In the context here, the aim is to estimate a lower or an upper limit value L so that at least a proportion γ of the population is greater or smaller than L. For example, an upper limit value L with a specified proportion of $\gamma = 0.95$ describes a level at which 95% of the population lie below L. In statistics, such limit values L are often referred to as lower one-sided or upper one-sided tolerance intervals.

In the calculation of such tolerance intervals, three cases generally need to be distinguished [3]:

1. The random variable X is normally distributed with known mean μ and known standard deviation σ ($X \sim N(\mu, \sigma)$, where the symbol "~" is read as "is distributed as")
2. The random variable X is normally distributed with unknown mean μ but known standard deviation σ ($X \sim N(\bar{x}, \sigma)$)
3. The random variable X is normally distributed with unknown mean μ and unknown standard deviation σ ($X \sim N(\bar{x}, s)$)

The first case is trivial as it leads directly to the determination of the lower or upper quantile of the entire known population, characterized by population mean μ and population standard deviation σ. Nevertheless, it is used in this paper as an introductive example into the subject and to illustrate some basic statistical concepts. The second case which is of remarkable interest in many applications (e.g., quality control) was not accounted for in the European standardization and, hence, is omitted in this paper. In practice, the third case is by far the most interesting one as it reflects the situation most frequently encountered in empirical sciences : from a limited random sample $x_1,...,x_n$, sample mean $\overline{x}$ and sample standard deviation s are obtained and the aim is to estimate a limit value ℓ above or below which a specified proportion γ of the population lies with confidence $1-\alpha$. The third case forms the basis for the determination of the statistical factors presented in EN 14385 and therefore will be discussed in full detail.

3. CALCULATION OF TOLERANCE INTERVALS

The European standard EN 14385 instructs the user to apply the lognormal distribution in order to model the frequency properties of the random variable X. However, the following results are independent of the type of normal distribution (normal or lognormal distribution). For sake of simplicity, subsequently only the normal distribution will be considered. In order to avoid any confusion with positive and negative signs, the calculation of an upper tolerance interval will be discussed. Of course, the resulting statistical factors can be used in the same manner to calculate lower tolerance intervals which are of interest when strength properties are concerned.

3.1 Normally distributed random variable X with known μ and σ

As an introductive example, a normally distributed population with known mean μ and known standard deviation σ is considered. The upper one-sided tolerance interval L, below which the proportion γ of the distribution lies, is then determined by the equation

$$L = \mu + K_\gamma \cdot \sigma \tag{1}$$

In order to obtain the appropriate factor K_γ, the probability

$$Pr\left(L \le \mu + K_\gamma \cdot \sigma\right) = \gamma \tag{2a}$$

needs to be calculated. This equation is equivalent to

$$Pr\left(\frac{L-\mu}{\sigma} \le K_\gamma \right) = \gamma \tag{2b}$$

Since the quotient in eq. (2b) is a standardized normal random variable ($(L-\mu)/\sigma \sim N(0,1)$), the factor K_γ is obtained as the solution of the equation

$$N(0,1;\gamma) := \int_{-\infty}^{K_\gamma} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = \gamma \tag{2c}$$

The determination of the factor K_γ is equivalent to the calculation of the upper quantile $N(0,1;\gamma)$ of the standardized normal distribution. For example, setting in eq. (2c) the proportion $\gamma = 0.95$ and solving for the upper integration limit yields the well-known factor $K_\gamma = 1.645$.

3.2 Normally distributed random variable X with unknown μ and σ

In practice, the entire population characterized by mean μ and standard deviation σ is rarely known. Rather, a limited random sample $x_1, ..., x_n$ with sample mean $\bar{x}$ and standard deviation s is given. An upper limit value ℓ, below which a specified proportion γ of the random sample lies, may be obtained in an entirely analogous manner as described in section 3.1

$$\ell = \bar{x} + K_\gamma \cdot s \tag{3a}$$

Contrary hereto, the aim is now to perform the statistical inference from the random sample to a specified proportion of the population. In other words, we seek an estimate $\ell_{1-\alpha}$ for the unknown limit value L of the population below which the proportion γ lies with confidence $1-\alpha$

$$\ell_{1-\alpha} = \bar{x} + k \cdot s \tag{3b}$$

The estimate $\ell_{1-\alpha}$ shall be at least equal L with confidence $1-\alpha$. As probability, this is expressed in the following equation [1,2]

$$Pr\left(\ell_{1-\alpha} \ge L\right) = Pr\left(\bar{x} + k \cdot s \ge \mu + K_\gamma \cdot \sigma\right) = 1-\alpha \tag{4}$$

which allows the determination of the appropriate factor k. Equation (4) can be rearranged to read

$$Pr\left(\frac{-\overline{x}+\mu+K_\gamma\cdot\sigma}{s} \leq k \right) = 1-\alpha \tag{5}$$

In order to perform the calculation of this probability, it is necessary to know how the quotient on the left hand side of the inequality in eq. (5) is statistically distributed. To make that evident, the expression in brackets can be algebraically expanded in an appropriate manner by σ, $\sqrt{n}$ and $\sqrt{n-1}$ [1]

$$Pr\left(U \leq k\sqrt{n} \right) = Pr\left(\frac{-\frac{\overline{x}-\mu}{\sigma}\sqrt{n}+K_\gamma\sqrt{n}}{\sqrt{n-1}\,\frac{s}{\sigma}}\sqrt{n-1} \leq k\sqrt{n} \right) = 1-\alpha \tag{6}$$

Applying the theory of functions of multivariate random variables [3,4] and introducing degrees of freedom $f = n-1$, the quotient U in eq. (6) can be rewritten

$$U = \frac{-X+\delta}{Y}\sqrt{f} \tag{7}$$

It can be verified by the transformation of a univariate function of one random variable [3,4], that the random variable X in the numerator of eq. (7) is distributed as a standardized normal distribution $N(0,1)$ with the probability density

$$X = \frac{\overline{x}-\mu}{\sigma}\sqrt{n} \quad \sim \quad f_X(x) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{x^2}{2}} \tag{7a}$$

The random variable Y in the denominator of eq. (7) has a χ-distribution with $f = n-1$ degrees of freedom and with probability density

$$Y = \sqrt{f}\,\frac{s}{\sigma} \quad \sim \quad f_Y(y) = \frac{1}{2^{\frac{f}{2}-1}\,\Gamma\left(\frac{f}{2}\right)}\, y^{f-1}\, e^{-\frac{y^2}{2}} \tag{7b}$$

where $\Gamma(x) = \int_0^\infty e^{-t}\, t^{x-1}\, dt$ denotes the complete Euler Gamma function.

The constant $\delta = K_\gamma\sqrt{n}$ in eq. (7) is termed noncentrality parameter, where K_γ is determined as outlined in section 3.1.

According to a famous proof of statistics [4], the sampling functions $\overline{x}$ and s are stochastically independent. Consequently, the transformed random variables X and Y are stochastically independent, too. In this case, the joint probability density $f_{X,Y}(x,y)$ is therefore simply given as the product of the probability densities of X and Y

$$f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y) = \frac{1}{\sqrt{2^{f-1}\pi}\,\Gamma\left(\frac{f}{2}\right)}\, y^{f-1}\, e^{-\frac{1}{2}\left(x^2+y^2\right)} \tag{8}$$

Assuming a sample size of $n = 10$, in Fig. 1 the marginal probability densities $f_X(x)$ and $f_Y(y)$ (according to eqs. (7a) and (7b)) are plotted qualitatively along the x- and y-axes while the joint probability density $f_{X,Y}(x,y)$ (according to eq. (8)) is shown as contour plot; all these probability densities are displayed in dashed style. Orientating, the both expectation values of the respective marginal probability densities of X and Y as well as their point of intersection are plotted.

Now, the probability density of the random variable U according to eq. (7) is obtained by means of the theory of transforming a multivariate function with two random variables [3,4]. The multivariate transformation rule is given by the equation system (see eq. (7))

$$u = \frac{-x+\delta}{y}\sqrt{f}\,, \quad v = y \tag{9a}$$

The inverse of this transformation rule is found to be

$$x = -\frac{u\,v}{\sqrt{f}} + \delta = h_1(u,v)\,, \quad y = v = h_2(u,v) \tag{9b}$$

The transformed joint probability density $f_{U,V}(u,v)$ is then defined by the equation

$$f_{U,V}(u,v) = f_{X,Y}\left(h_1(u,v), h_2(u,v)\right) \cdot \left|\det \mathbf{J}\right| \tag{10}$$

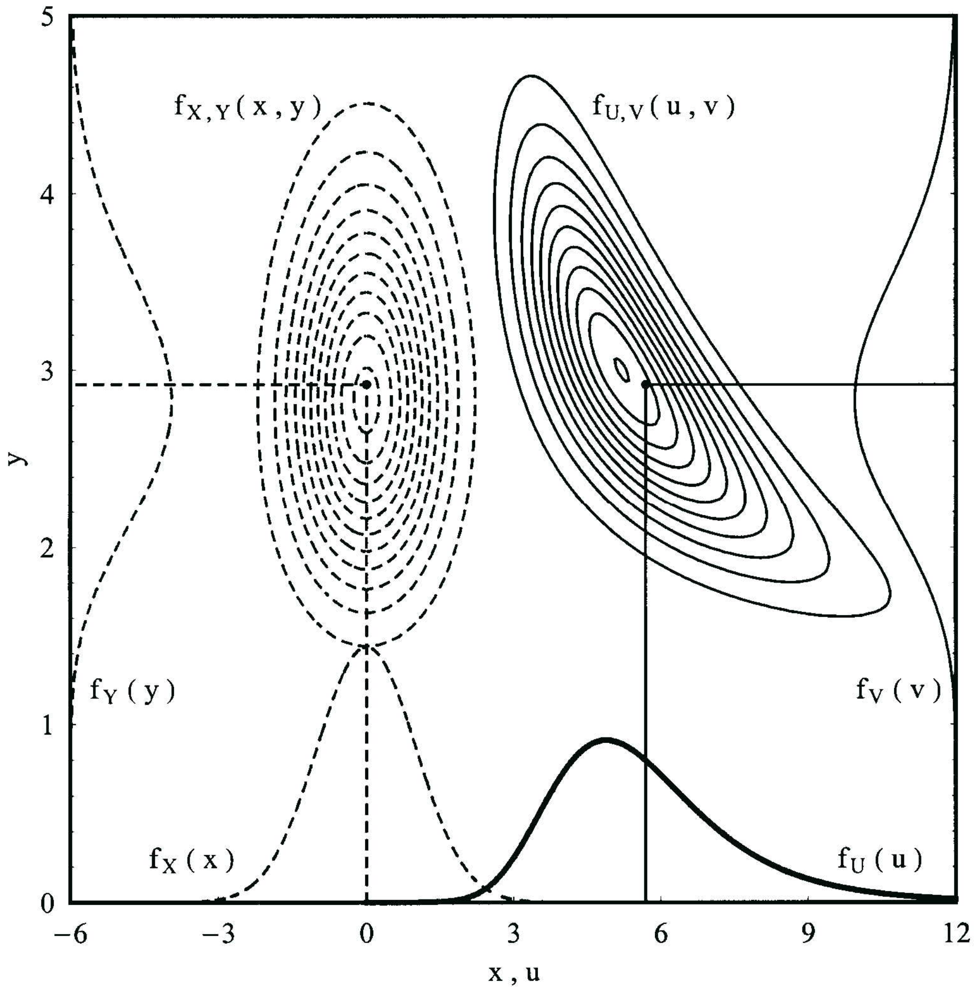

Fig. 1 : *Marginal probability densities $f_X(x)$ and $f_Y(y)$, expectation values and joint probability density $f_{X,Y}(x,y)$ (dashed style). Joint probability density $f_{U,V}(u,v)$, marginal probability densities $f_U(u) = f_t(t;f,\delta)$ and $f_V(v) = f_Y(y)$ and expectation values (solid style)*

where | det **J** | is the determinant of the Jacobi matrix. For the given case, the determinant of the Jacobi matrix results in

$$\left|\det \mathbf{J}\right| = \left|\det\begin{pmatrix} \dfrac{\partial h_1(u,v)}{\partial u} & \dfrac{\partial h_1(u,v)}{\partial v} \\ \dfrac{\partial h_2(u,v)}{\partial u} & \dfrac{\partial h_2(u,v)}{\partial v} \end{pmatrix}\right| = \left|\det\begin{pmatrix} -\dfrac{v}{\sqrt{f}} & -\dfrac{u}{\sqrt{f}} \\ 0 & 1 \end{pmatrix}\right| = \frac{v}{\sqrt{f}} \qquad (11)$$

Inserting eqs. (9b) and (11) in eq. (10) yields the transformed joint probability density

$$f_{U,V}(u,v)=\frac{1}{\sqrt{2^{f-1}\pi\, f}\;\Gamma\left(\frac{f}{2}\right)}\, v^{f}\, e^{-\frac{1}{2}\left[\left(-\frac{u\,v}{\sqrt{f}}+\delta\right)^{2}+v^{2}\right]} \tag{12}$$

Assuming again a sample size of $n=10$ and $K_\gamma = 1.645$ corresponding to a proportion $\gamma = 0.95$ (noncentrality parameter $\delta = K_\gamma \sqrt{n} = 1.645 \cdot \sqrt{10} = 5.202$), the transformed joint probability density $f_{U,V}(u,v)$ is shown as contour plot with solid lines in Fig. 1, as well.

The probability density of the random variable U according to eq. (7) is now obtained as the marginal probability density $f_U(u)$ of $f_{U,V}(u,v)$

$$f_U(u)=\int_0^{\infty} f_{U,V}(u,v)\,dv \tag{13}$$

Performing this integration yields the probability density of what is referred to as noncentral *t*-distribution. The probability density exists in analytic form and is found to be [3]

$$f_U(u)=f_t(t;f,\delta)=\frac{e^{-\frac{\delta^2}{2}}\, f^{\frac{f}{2}}}{\sqrt{\pi}\;\Gamma\left(\frac{f}{2}\right)\left(f+t^2\right)^{\frac{f+1}{2}}}\sum_{j=0}^{\infty}\frac{\Gamma\left(\frac{f+j+1}{2}\right)}{j!}\left[\frac{t\,\delta\sqrt{2}}{\sqrt{f+t^2}}\right]^{j} \tag{14}$$

with parameters $f = n-1$ (degrees of freedom) and $\delta = K_\gamma\sqrt{n}$ (noncentrality parameter). Equation (7) in connection with eqs. (7a, b) are called the *constructive definition*, since these equations define the noncentral *t*-distribution in an unique manner.

Under the assumptions given above ($n=10, \delta = 5.202$), the marginal probability density $f_U(u) = f_t(t;f,\delta) = t(f,\delta)$ is plotted qualitatively in Fig. 1 along the u-axis in fat solid style. The second marginal probability density $f_V(v)$ is plotted qualitatively along the v-axis. Note, that $f_V(v) = f_Y(y)$ since the transformation rule $v = y$ (see eq. (9a)) for this coordinate is just an identity and involves no transformation. Orientating, the both expectation values of the respective marginal probability densities of U and V as well as their point of intersection are plotted in Fig. 1, again.

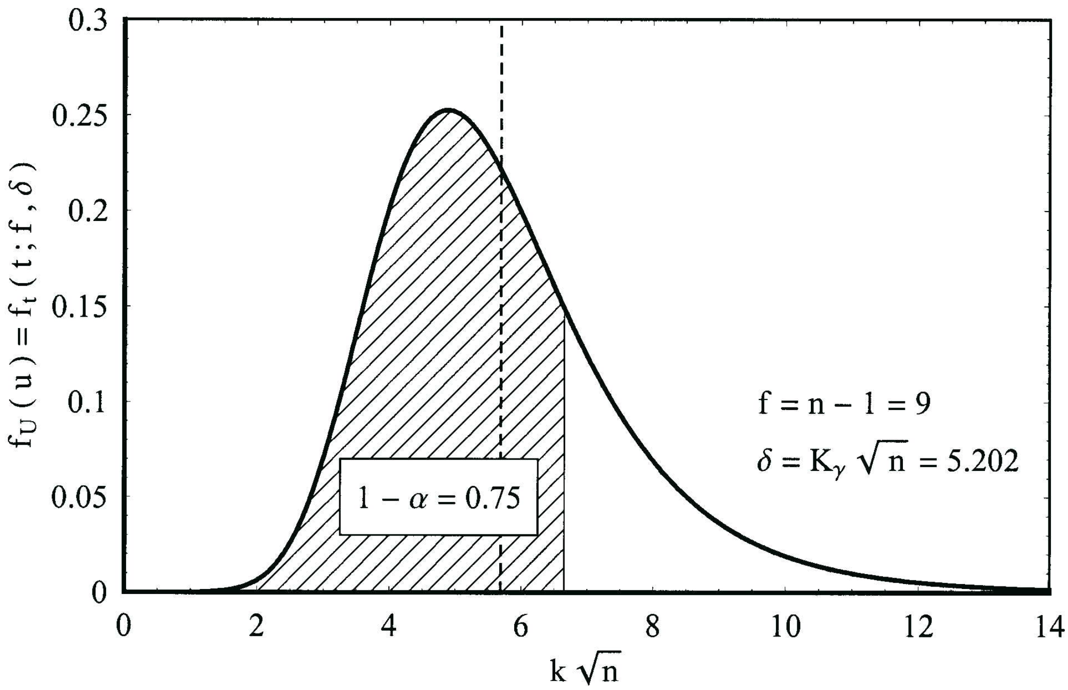

Fig. 2 : *Probability density* $f_U(u) = f_t(t; f, \delta)$, *expectation value (dashed vertical line) and confidence level* $1 - \alpha = 0.75$ *(hatched area)*

So far it is proven that the expression $k\sqrt{n}$ according to eq. (6) is distributed as a noncentral *t*-distribution with probability density

$$k\sqrt{n} \sim t\,(f, \delta) \tag{15}$$

with $f = n - 1$ degrees of freedom and $\delta = K_\gamma\sqrt{n}$ as noncentrality parameter. The quantity K_γ is obtained according to eq. (2c), whereas γ denotes the proportion of the population specified to be at least below the upper limit value $\ell_{1-\alpha}$ with confidence $1 - \alpha$. Again assuming $n = 10$ and $\delta = 5.202$, the probability density of $k\sqrt{n}$ is plotted in Fig. 2.

Now, it is easy to evaluate the probability $Pr\left(U \le k\sqrt{n}\right) = 1 - \alpha$ as given in eq. (6) numerically in dependence of the sample size n and of the proportion γ. In mathematical terms, this is accomplished by solving the probability integral

$$t(f, \delta; 1 - \alpha) := \int_{-\infty}^{k\sqrt{n}} f_t(t; f, \delta)\, dt = 1 - \alpha \tag{16}$$

for the upper integration limit $k\sqrt{n}$. The calculation performed in eq. (16) is equivalent to the determination of an upper one-sided confidence interval for the quantity $k\sqrt{n}$ with confidence $1-\alpha$. Figure 2 shows graphically the meaning of this confidence interval where the confidence level is displayed as hatched area (assumed confidence level : $1-\alpha=0.75$). Orientating, again the expectation value of the probability density is plotted as a dashed vertical line.

From eq. (16) immediately follows that the factor k is obtained as [1]

$$k_{\gamma,n,1\text{-}\alpha} = \frac{t\left(f,\delta;1-\alpha\right)}{\sqrt{n}} = \frac{t\left(f,K_{\gamma}\sqrt{n};1-\alpha\right)}{\sqrt{n}} \tag{17}$$

It can be seen that in the case of unknown mean μ and unknown standard deviation σ the factor k according to eq. (3b) becomes a function of the proportion γ, the sample size n and the confidence level $1-\alpha$. Inserting eq. (17) into eq. (3b) yields the final equation for an upper one-sided tolerance interval

$$\ell_{1-\alpha} = \overline{x} + k_{\gamma,n,1-\alpha} \cdot s = \overline{x} + t\left(f,\delta;\,1-\alpha\right)\cdot\frac{s}{\sqrt{n}} \tag{18}$$

A lower one-sided tolerance interval ℓ_{α}, as it is of interest for strength properties, is obtained by inserting the quantile $t\left(f,-\delta;\,\alpha\right)$ into eq. (18) instead of $t\left(f,\,\delta;\,1-\alpha\right)$ which results in negative but numerically identical values.

It is interesting to note that, if the noncentrality parameter is set $\delta=0$, eq. (18) becomes identical to the equation for the determination of an upper one-sided confidence interval for the (unknown) population mean μ (50%-quantile). In this case, the noncentral *t*-distribution simplifies to the more familiar central (Student) *t*-distribution $t\left(f,0\right)=t\left(f\right)$ with the upper one-sided quantile $t\left(f;\,1-\alpha\right)$. Therefore, tolerance intervals as discussed in this section must be regarded as one-sided confidence intervals for the quantiles of a normal distribution $N\left(\overline{x},\,s\right)$.

This result is in contradiction with the terminology of EN 14358 where it is stated that the "5%-quantile" is the result of the evaluation yet, more specifically speaking, one-sided confidence intervals for the quantiles of a normal distribution with unknown mean μ and unknown standard deviation σ are obtained.

4. EVALUATION OF THE STATISTICAL FACTORS

As stated in the introduction, the tables given in EN 14358 are rather incomplete. Only a few statistical factors are given in dependence of sample size. As the gaps between the different sample sizes are rather large, the interpolated statistical factors become correspondingly inaccurate. It is hence desirable, to have tables with numerically precise figures at hand. These tables are the subject of this chapter.

In EN 14358, the proportion as understood in the previous chapters is set $\gamma = 0.95$ while two confidence levels are proposed for the application : $1-\alpha = 0.75$ and $1-\alpha = 0.841$. The latter confidence level corresponds to the integral of the standardized normal probability density running from negative infinity to one.

Having the proportion γ and the confidence levels $1-\alpha$ defined it is a simple task to evaluate the statistical factors $k_{\gamma,n,1-\alpha}$ according to eqs. (16) and (17). For the confidence level $1-\alpha = 0.75$ the factors $k_{\gamma,n,1-\alpha}$ are listed in Table 1 and for the confidence level $1-\alpha = 0.841$ in Table 2.

5. CONCLUSIONS

In the presented paper, the determination of characteristic values for structural timber based on the European standard EN 14358 was considered. A comprehensive derivation of the statistical factors given in this standard was performed by means of mathematical statistics in order to illustrate the assumptions and the background of these figures. It was found that the terminology of EN 14358 is imprecise in so far as the “5%-quantile” is termed to be the result whereas actually a lower one-sided confidence interval for the 5%-quantile is obtained as result. For a more convenient application of EN 14358, complete tables with statistical factors were provided based on the two confidence levels proposed in the code.

Table 1

Statistical factors $k_{\gamma,n,1-\alpha}$ for the calculation of one-sided tolerance intervals

(Population mean μ and standard deviation σ unknown)

Proportion : $\gamma = 0.95$, confidence level : $1 - \alpha = 0.75$

n	$k_{0.95,n,0.75}$	n	$k_{0.95,n,0.75}$	n	$k_{0.95,n,0.75}$	n	$k_{0.95,n,0.75}$
1	-	26	1.889	51	1.809	76	1.776
2	5.122	27	1.883	52	1.807	77	1.775
3	3.152	28	1.878	53	1.805	78	1.774
4	2.681	29	1.873	54	1.804	79	1.773
5	2.463	30	1.869	55	1.802	80	1.772
6	2.336	31	1.864	56	1.801	81	1.771
7	2.250	32	1.860	57	1.799	82	1.771
8	2.188	33	1.856	58	1.797	83	1.770
9	2.141	34	1.853	59	1.796	84	1.769
10	2.104	35	1.849	60	1.795	85	1.768
11	2.073	36	1.846	61	1.793	86	1.767
12	2.048	37	1.842	62	1.792	87	1.767
13	2.026	38	1.839	63	1.791	88	1.766
14	2.007	39	1.836	64	1.789	89	1.765
15	1.991	40	1.834	65	1.788	90	1.764
16	1.976	41	1.831	66	1.787	91	1.764
17	1.963	42	1.828	67	1.786	92	1.763
18	1.952	43	1.826	68	1.784	93	1.762
19	1.941	44	1.824	69	1.783	94	1.762
20	1.932	45	1.821	70	1.782	95	1.761
21	1.923	46	1.819	71	1.781	96	1.760
22	1.915	47	1.817	72	1.780	97	1.760
23	1.908	48	1.815	73	1.779	98	1.759
24	1.901	49	1.813	74	1.778	99	1.758
25	1.895	50	1.811	75	1.777	100	1.758

Table 2

Statistical factors $k_{\gamma,n,1-\alpha}$ for the calculation of one-sided tolerance intervals

(Population mean μ and standard deviation σ unknown)

Proportion : $\gamma = 0.95$, confidence level : $1 - \alpha = 0.841$

n	$k_{0.95,n,0.841}$	n	$k_{0.95,n,0.841}$	n	$k_{0.95,n,0.841}$	n	$k_{0.95,n,0.841}$
1	-	26	2.009	51	1.889	76	1.840
2	8.199	27	2.001	52	1.886	77	1.838
3	4.118	28	1.993	53	1.884	78	1.837
4	3.281	29	1.986	54	1.881	79	1.836
5	2.915	30	1.979	55	1.879	80	1.834
6	2.706	31	1.972	56	1.876	81	1.833
7	2.569	32	1.966	57	1.874	82	1.832
8	2.471	33	1.960	58	1.872	83	1.830
9	2.397	34	1.954	59	1.870	84	1.829
10	2.338	35	1.949	60	1.867	85	1.828
11	2.291	36	1.944	61	1.865	86	1.827
12	2.251	37	1.939	62	1.863	87	1.826
13	2.218	38	1.935	63	1.861	88	1.825
14	2.189	39	1.930	64	1.860	89	1.823
15	2.164	40	1.926	65	1.858	90	1.822
16	2.142	41	1.922	66	1.856	91	1.821
17	2.122	42	1.918	67	1.854	92	1.820
18	2.105	43	1.914	68	1.852	93	1.819
19	2.089	44	1.911	69	1.851	94	1.818
20	2.074	45	1.907	70	1.849	95	1.817
21	2.061	46	1.904	71	1.847	96	1.816
22	2.049	47	1.901	72	1.846	97	1.815
23	2.038	48	1.898	73	1.844	98	1.814
24	2.028	49	1.895	74	1.843	99	1.813
25	2.018	50	1.892	75	1.841	100	1.812

Of course, the outlined evaluation procedure is not limited to the determination of characteristic values for structural timber. Rather, the procedure is of general validity. As statistics is an exact science, there can be one and only one way to determine one-sided confidence intervals for the quantiles of a normally distributed random variable. The more surprising it is that other European standards (e.g., EN 1058 for wood based panels [6]) propose evaluation procedures which totally deviate from the one discussed here. The validity of these deviating procedures will be investigated in following papers.

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