

DETERMINATION OF LOCAL AND GLOBAL MODULUS OF ELASTICITY IN WOODEN BOARDS

BESTIMMUNG DES LOKALEN UND GLOBALEN ELASTIZITÄTSMODUL IN HOLZBRETTERN

DETERMINATION DU MODULE D'ELASTICITE LOCAL ET GLOBAL SUR DES PANNEAUX A BASE DE BOIS

Simon Aicher, Lilian Höfflin, Wolfgang Behrens

SUMMARY

The paper reports on an efficient method for determination of the local modulus of elasticity by means of elongation/strain measurements. Further, the effect of local weak sections on the global modulus of elasticity determined by deflection measurement is revealed. The global modulus of elasticity computed on the basis of the partly extremely varying locally measured moduli of elasticity complies well with the globally measured MOE.

The experimental investigations were performed with edgewise bent beech boards. First, the elongation /strain measurement method was verified exemplarily with a board which was inflicted successively with artificial defects (holes). For each defect state the local and global moduli of elasticity were measured and the differences are discussed. Second, the variation of local modulus of elasticity and its high spatial correlation with the location of bending failure is shown exemplarily by means of four beech boards of a larger test series.

ZUSAMMENFASSUNG

Es wird über eine effiziente Methode zur Bestimmung des lokalen Elastizitätsmoduls mittels Längsverschiebungs-/Dehnungsmessungen berichtet. Desweiteren wird die Auswirkung lokaler Schwachstellen auf den mittels Durchbiegungsmessung bestimmten globalen Elastizitätsmodul gezeigt. Der aus den teilweise extrem variierenden lokal gemessenen Elastizitätsmoduln berechnete globale Elastizitätsmodul stimmt sehr gut mit dem gemessenen globalen Elastizitätsmodul überein.

Die experimentellen Untersuchungen wurden mit hochkant biegebeanspruchten Buchebrettern durchgeführt. Zuerst wurde die Längsverschiebungs-/Dehnungsmeßmethode exemplarisch an einem Brett verifiziert, welches stufenweise mit künstlichen Defekten (Löchern) versehen wurde. Für jeden Defektzustand wurden die lokalen und globalen Elastizitätsmoduln gemessen. In einem zweiten Schritt wird die Änderung des lokalen Elastizitätsmoduls und dessen hohe Korrelation mit dem Ort des Biegeversagens exemplarisch an vier Brettern einer größeren Versuchsreihe aufgezeigt.

RESUME

Cet article présente une méthode efficace permettant de déterminer le module d'élasticité local par une mesure couplée déplacement/déformation. D'autre part, on fait apparaître l'effet des sections localement faibles sur le module d'élasticité global déterminé par la flèche. Le module d'élasticité global obtenu par intégration des modules locaux mesurés, extrêmement variables, est en bon accord avec le module global mesuré.

L'étude expérimentale a porté sur des panneaux fléchis à chant. La méthode couplée déplacement/déformation a été préalablement vérifiée sur un panneau présentant des défauts artificiels (trous). Pour chaque défaut, on détermine les modules local et global, et les différences sont discutées. Par la suite, la variation du module local et sa forte corrélation spatiale avec la résistance en flexion est mise en évidence sur 4 panneaux de hêtre extraits d'une campagne expérimentale plus importante

KEYWORDS: local modulus of elasticity, global modulus of elasticity, stiffness variation, artificial defects, weak sections

1 INTRODUCTION

It is reported on a method for determination of the local modulus of elasticity (MOE) in bending tests with timber beams and respective results. In heterogeneous materials such as wood the modulus of elasticity can vary strongly along the length of the boards. Based on a positive stiffness - bending strength correlation, the footprints of locally low MOE values determine the strength class (or grade) of boards in grading machines based on the bending principle. Local MOE obviously depends strongly on the length of the board segment used

for the MOE determination which in most cases is larger than a local weak area, mostly created by a knot or by sloping grain.

A local MOE determined over a board segment length of 5 times the cross-sectional depth as specified in EN 408 still represents an integral (constant) value over a considerable length and there may be strong local MOE deviations within that length. The stated averaging effect of concentrated zones of low MOE areas occurs in all bending type grading machines which bend at consecutive locations, as there are practical limits for the length of the span. This is the major reason for the moderate coefficient of correlation between bending strength and MOE. Interesting approaches on how to reconstruct the variation of the true MOE function from MOE data collected from a consecutively bent board in order to derive the true local MOEs based on Fourier transforms were proposed by Bechtel (1985) and Foschi (1987).

Apart from strength grading the knowledge of the actual local MOE and of the associated local strength variation along the length of boards is very important for (stochastic) modelling of boards and glulam subdivided in unit cells of small length, i.a. Foschi and Barrett (1980), Ehlbeck et al. (1985), Isaksson (1999) and Serrano (2001). Hereby the length of the unit cell has a considerable modelling influence on load sharing in adjacent glulam lamellas.

For modelling and calibrating the MOE variation along board length several approaches are known (i.a. Foschi and Barrett (1981), Ehlbeck et al. (1985), Kline et al. (1986) and Taylor (1991)). All models are based on a calibration vs. global (and partly local) modulus of elasticity necessitating extensive empiric data and leaving model dependent considerable uncertainties.

The experimental determination of local MOEs which at first view seems to be a very simple task is demanding in case a bending method is applied and has limits concerning the smallness of the segment length. Reliable results below span to depth ratios of about 3 are questionable; limits were revealed by Kaas (1975) employing the so-termed "middle ordinate method". The method is based on the assumption that short segments of a bent board approximate arcs of circles with varying radii.

The work reported here was conducted in the frame of establishing a realistic empirical data basis for the variation of bending MOE and bending strength values along the length of beech wood boards bent about the major axis. It was

intended to measure the local bending MOE over distances or unit cell lengths in the range of 1 to 2 times of the depth of the boards.

The paper shows a method for determination of the local modulus of elasticity by means of displacement/strain measurements and reveals the effect of local weak sections in comparison to the global modulus of elasticity determined by deflection measurement. The experimental verification of the method which in principle is independent of specific materials is performed with edgewise loaded beech boards. First, the displacement/strain measurement method is verified exemplary with a board which was inflicted successively with artificial defects. For each defect state the local and global modulus of elasticity were measured and the differences are discussed. Finally, the variation of local modulus of elasticity and its local correlation with the position of failure is shown exemplarily on four beech boards of a larger test series.

2 GLOBAL AND LOCAL METHOD FOR DETERMINATION OF MODULUS OF ELASTICITY IN BENDING

It is important to note that the terms “local MOE” and “global MOE” here are defined different as in European standard EN 408. In the mentioned standard the so-termed local MOE is determined in a 4point bending test with loads in the 3rd points via deflection measurement within the constant moment length of 6 times the depth h of the beam. The deflection w_1 is actually determined over a length of $\ell_m = 5 h$ (Fig. 1a). Contrary thereto, so-termed “global MOE” is determined acc. to EN 408 from deflection measurement w_2 over full span of 18 h including effects of shear and of indentations at the support locations (Fig. 1a).

In this paper global MOE is determined similar as local MOE acc. to EN 408 via deflection measurement w_1 within an enlarged constant moment area of 9 h . Local MOE is determined by elongation/strain measurement over the length of a small segment of the beam (see below).

2.1 Global modulus of elasticity

Global MOE based on deflection w_1 in the constant moment area is (see Fig. 1a)

$$E_{\text{glob}} = \frac{F \ell_a \ell_m^2}{8 I w_1} \quad (1)$$

with I = moment of inertia vs. major axis.

This measurement delivers an integral value over the length of ℓ_m .

2.2 Local modulus of elasticity

For determination of the local modulus of elasticity the bending compliance of the beam was no more determined by deflection measurement but instead by local elongation/strain measurements. The employed measuring principle is illustrated in Fig. 1b. The method consists of elongation/strain measurements over “small” lengths at the bending tension and compression edge of the beam. Based on the strains of the segment, ε_c and ε_t , the curvature κ of the segment of length L is given by (h = beam depth)

$$\kappa = \frac{(|\varepsilon_c| + \varepsilon_t)}{h} \quad \text{where} \quad \varepsilon_{c(t)} = \frac{\Delta u_{c(t)}}{L}. \quad (2a, b)$$

The bending MOE of the segment is then obtained from the curvature-moment relationship

$$E_{\text{seg}} = \frac{M}{\kappa I} \quad \text{with} \quad M = F \ell_a \quad (3)$$

The introduced elongation/strain measurement method is limited to small deflections; the arc length of the bending line of the beam segment has to be approximately equal to its chord length. This is the case for small ratios of L/h . Here length L was chosen with respect to modelling aspects (smallest employed “cell” size), not discussed in this paper, in conjunction with existing equipment as $L = 200$ mm. This is roughly 1.5 times the depth of the investigated beams. The determination of the elongations can be performed very accurately, say reproducible, by means of so-called on-set strain extensometers. Despite the misleading term “strain extensometer” the measurement actually represents an elongation measurement of an exactly determined base length L . In order to establish the base length, small fixing plates ($\varnothing 8$ mm) with a conical hole for the pin pointed extensometer legs are glued (wax) to the measured object, here to the narrow faces of the beam.

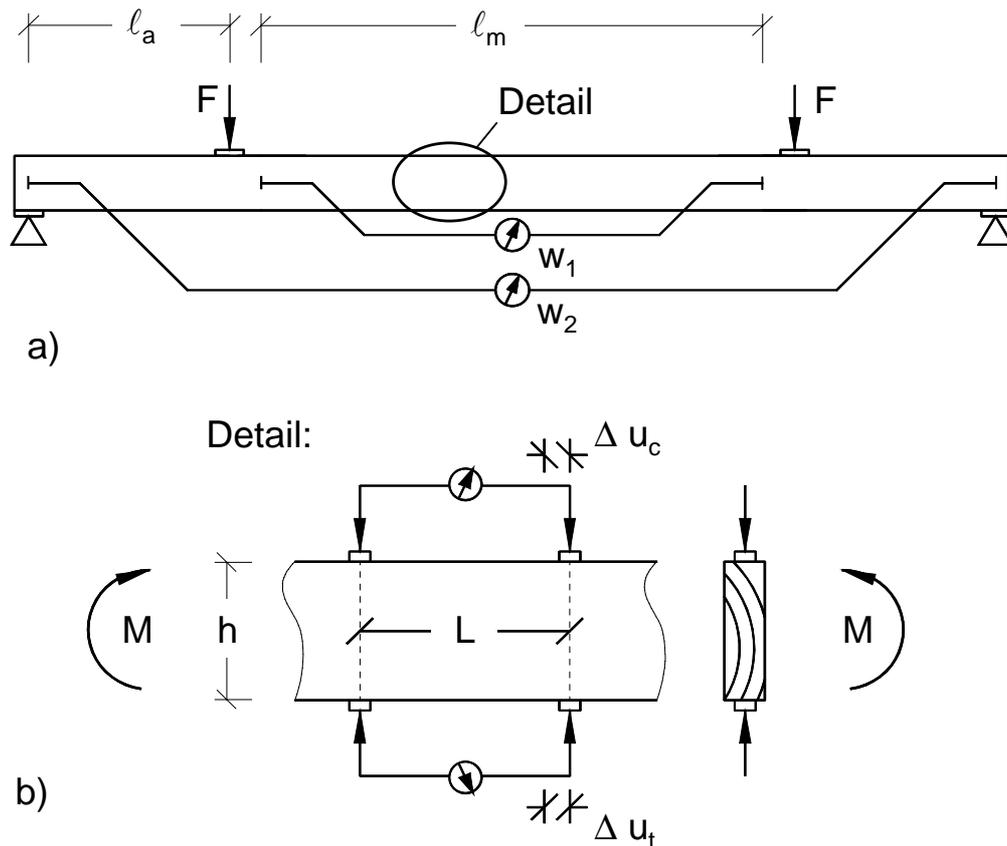


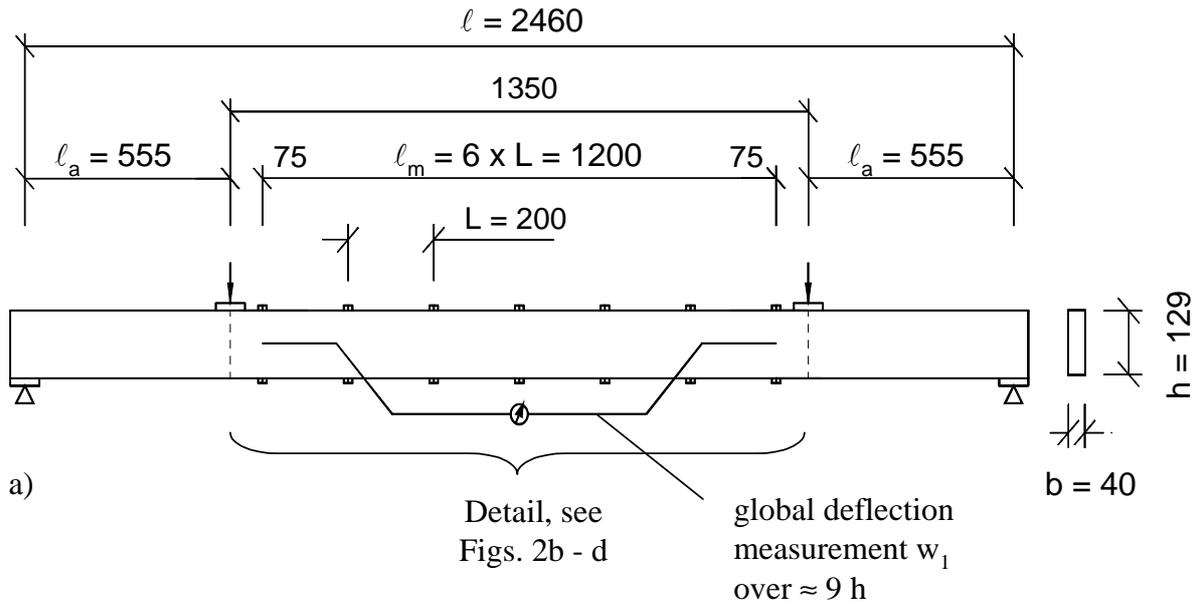
Fig. 1: Schematic illustration of global and local MOE determination

- a) global MOE based on deflection measurement of w_1 within constant moment area (Note: this is different from EN 408 where global MOE is related to deflection w_2)
- b) local MOE based on elongation/strain measurement at top and bottom edge of the beam over a small length

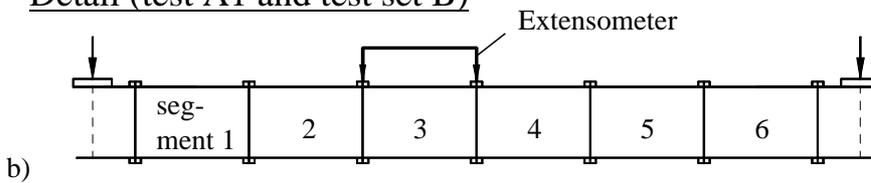
3 TEST CONFIGURATION AND PROGRAM

The reported investigations comprise two test sets A and B of experiments, all with beam specimens of equal size and loading. In test set A the differences of the specifically employed local and global MOE determination were regarded in detail, in test set B the (spatial) correlation of local MOE and the location of bending failure was investigated.

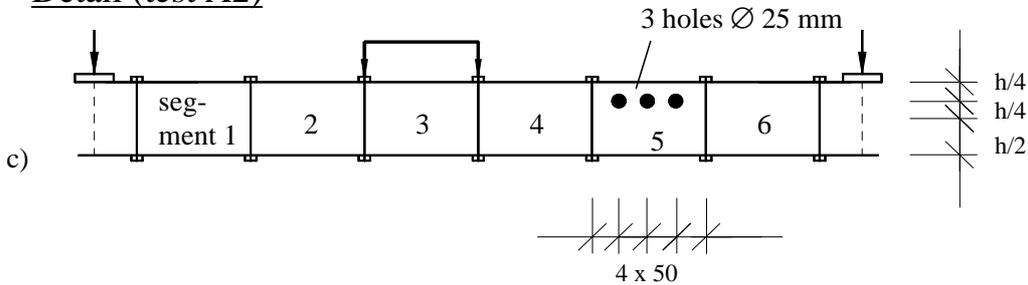
All investigations were performed as 4point bending tests with span to depth ratio of about 19 h. The constant moment length was chosen fairly large as about 9 times the depth of the board (see Fig. 2). The constant moment length was then divided into six equal sized segments of $L = 200$ mm, which is approximately 1.5 times the beam depth. For each segment the local modulus of elasticity was determined. Additionally the global modulus of elasticity was



Detail (test A1 and test set B)



Detail (test A2)



Detail (test A3)

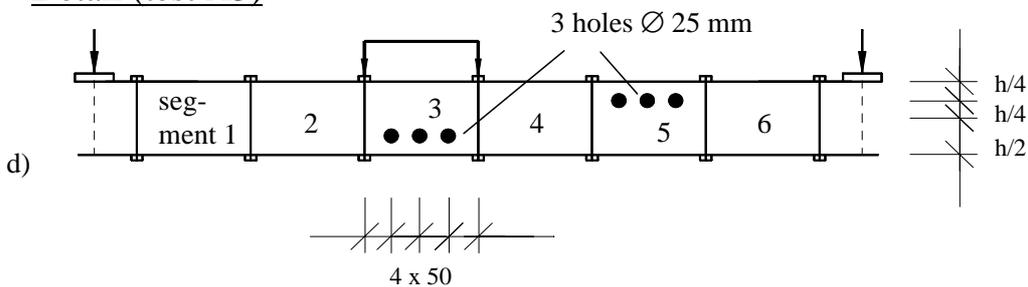


Fig. 2a-d: investigated specimen and loading configurations in test sets A and B

- a) general test set-up
- b) test A1 (with board No. 1) and test set B; only natural defects (if at all) in segments 1 – 6
- c) test A2; 3 artificial holes in segment 5 of board No. 1
- d) test A3; 3 additional artificial holes in segment 3 of board No. 1

determined via deflection measurement over the length $\ell_m = 6 L$. A distance of 75 mm between the load application points and the outer segments was chosen in order to avoid strain disturbances due to the load concentration. The distance between support and load application was 4 times the board depth in order to avoid shear failure in test set B.

The elongation has been measured with a “strain” extensometer with a resolution of 0.01 mm. Each segment length L has been measured two times, first in the unloaded state and second in the loaded state at 3 kNm ($\sigma_m = 27 \text{ N/mm}^2$), being roughly 1/3 of the mean failure moment.

Test set A: In order to prove for the employed local MOE method in an exemplary manner the ability to reveal the pronounced effect of local MOE variation, first three tests A1, A2 and A3 were performed with one board (No. 1) which was inflicted in tests A2 and A3 with artificial defects. All tests comprised the six local and one global MOE measurements. In detail, in test A1 the native board, being free of knots, was investigated (see Fig. 2b). In test A2 three holes all with a diameter of 25 mm were drilled into beam segment 5 close to the bending compression edge (see Fig. 2c). In test A3 three additional holes ($\varnothing 25$ mm) were drilled into beam segment 3, now close to the bending tension edge (see Fig. 2d).

Test set B: In on-going tests the described local and global MOE measurements were applied so far to 30 boards loaded to failure after compliance determination at intermediate load stop at 3 kNm. One result evaluation reported here was related to the spatial correlation of the failure location with the local MOE distribution.

4 RESULTS OF TEST SET A

Tables 1 and 2 contain the results of the tests A1 to A3, i.e. the local strains and the MOEs of the six segments and the global MOE. In addition, Tab. 2 contains finite element calculated global MOEs ($E_{\text{glob,calc}}$) based on the experimentally determined local MOEs (E_{seg}). The theoretical global MOE determined as in the experiment from the global deflection in the constant moment area, serves as a plausibility control of both, the locally and globally determined MOE. Figures 3 to 5 give a graphical representation of the strain and MOE results. Hereby, the local MOEs are given as constant values within the specific segment

whereas the strains of the segments are shown for the center of the segment allowing a better visual differentiation of strains and MOEs.

In all three tests A1 to A3 throughout a very good agreement between the experimentally and computationally obtained global MOEs was observed. The deviation was maximally 2%. Following the results are discussed in detail:

Test A1: Figure 3 reveals a very moderate local MOE variation (13350 to 14420 N/mm²) around the constant global value (13920 N/mm²). The extreme deviations of the local MOEs (E_{seg}) vs. E_{glob} are + 3.6% and – 4.2%.

Test A2: Figure 4 shows a pronounced drop of the local MOE in segment 5 where the holes were placed. The difference between the extreme local MOEs along span is 19%; the extreme deviations of the local MOEs vs. E_{glob} now are + 8.6% and – 9.1%. The effect of the local stiffness decrease on the global MOE, however, is still moderate; E_{glob} now is 4.2% less compared to test A1 whereas E_{seg5} decreased by 13%. It should be noted that the measured strains show clearly the position of the defect application (see also Tab. 1). Whereas strain ϵ_t at the bending tension edge of segment 5 is almost unchanged as compared to the measurement in test A1, strain ϵ_c at the bending compression edge of segment 5 shows a pronounced increase of 30%. In all other segments the local strains remain very similar to those measured in test A1. In this context it should be stated that the employed local strain based MOE determination gives an error of maximally ± 300 N/mm² at repeated measurements in conjunction with the used extensometer.

Test A3: It can be seen from Fig. 5 that the additional artificial defects (equal sizes and numbers as in test A2) applied close to the bending tension edge in segment 3 lead to a pronounced decrease of E_{seg3} of 14% vs. the former value in test A2. The obtained reduction of local MOE resembles very closely the decrease of E_{seg5} in test A2. The difference between the extreme local MOEs along ℓ_m is 14%; the extreme deviations of the local MOEs vs. E_{glob} now are + 12.4% and –2.9%. This second set of defects now reduces the global MOE by 3.5% compared to the A2 result. Again the strains clearly show the depth location of the new defect; now the strain at the bending tension edge increases by 24%, whereas the strain at the bending compression edge of segment 3 remains rather unchanged. The largest difference between local and global MOE now increased to 12.4%.

Table 1: Compilation of local compression and tension strains of test set A

test	compression and tension strain per 1kNm		local strains in segments 1 to 6					
			1	2	3	4	5	6
A1	ϵ_c	10^{-5}	-62.4	-60.4	-58.7	-64.9	-61.3	-60.2
	ϵ_t	10^{-5}	71.2	66.8	66.7	70.2	67.5	64.9
A2	ϵ_c	10^{-5}	-63.3	-59.5	-57.8	-66.7	-79.4	-62.9
	ϵ_t	10^{-5}	70.3	66.8	66.6	70.2	68.4	64.9
A3	ϵ_c	10^{-5}	-61.5	-57.7	-61.4	-66.7	-74.0	-61.1
	ϵ_t	10^{-5}	70.3	66.8	82.8	72.0	68.4	64.9

Table 2: Compilation of results for local and global MOEs of test set A

test	measured local MOEs in segments 1 to 6						global measured MOE	calculated MOE based on the measured local MOEs
	E_{seg1}	E_{seg2}	E_{seg3}	E_{seg4}	E_{seg5}	E_{seg6}	E_{glob}	$E_{glob, calc}$
-	N/mm ²	N/mm ²	N/mm ²	N/mm ²	N/mm ²	N/mm ²	N/mm ²	N/mm ²
A1	13495	14172	14381	13347	13989	14415	13919	13932
A2	13494	14272	14493	13171	12197	14110	13340	13576
A3	13686	14479	12502	13000	12660	14311	12879	13141

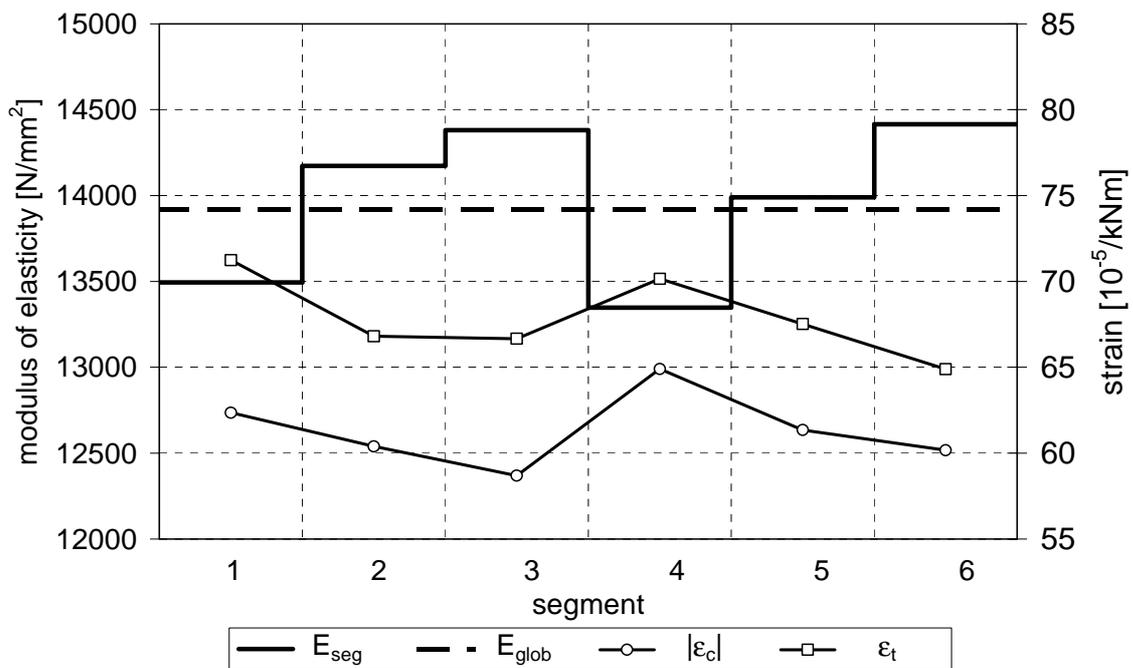


Fig. 3: Local strains, local and global MOEs of test A1 with board No. 1; no artificial defects

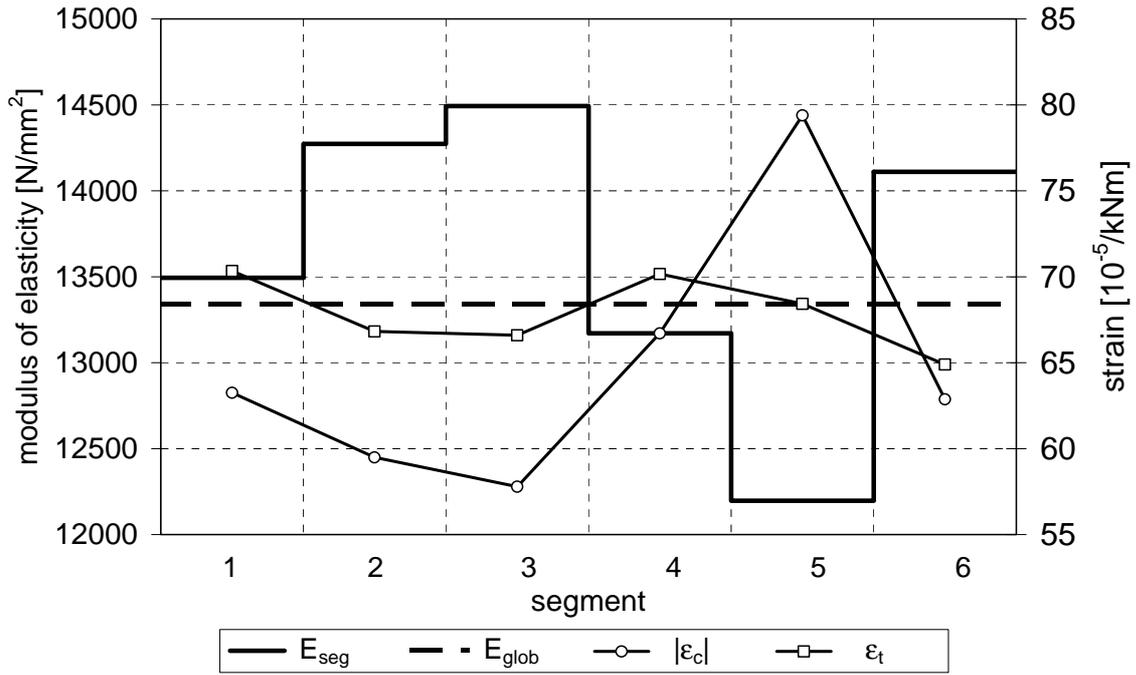


Fig. 4: Local strains, local and global MOEs of test A2 with board No. 1; artificial defects in the bending compression part of segment 5

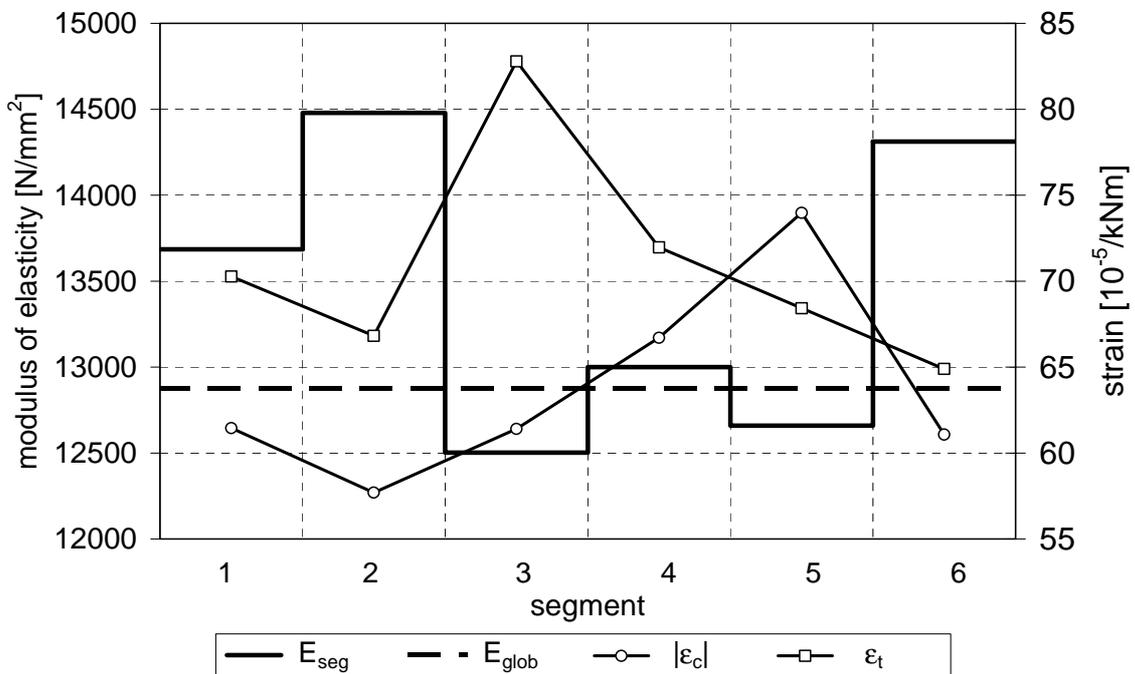


Fig. 5: Local strains, local and global MOEs of test A3 with board No. 1; additional artificial defects in the bending tension part of segment 3

5 RESULTS OF TEST SET B

Table 3 specifies the measured local and global MOEs, the computed global MOEs and the location of the failure of four beech boards. The chosen examples are exemplary for 30 tests performed so far. Figures 6 to 9 give graphical representations of the results including the local strain variations.

Figure 6 reveals the case of a nearly homogeneous board with no knots and no apparent grain deviation. Local and global MOEs show very little differences. Despite the small stiffness variations, the bending tension failure occurred in segment 3 with the lowest local MOE and with highest tension strain. The minimum local MOE differed only by 3% from the global MOE and only by 1.3% from the next weakest segment 1.

Figure 7 also depicts the strains and local MOE variations of a board without knots, but nevertheless with pronounced differences (maximally 18%) of local MOEs ranging from 12600 to 15400 N/mm². The extreme deviations of the local MOEs vs. the global MOE are + 7.7% and – 11.8%. The strains of segments 2 and 3 with lowest local MOEs show an interesting feature being that maximum tension and compression strain occur successively in segments 2 and 3, indicating sloping grain. The specimen failed in bending tension at the transition of segments 2 and 3.

Figure 8 relates to a board with a knot of 22 mm diameter and associated strong fibre deviations around the knot located in the upper bending compression part of segment 3. The very pronounced difference between the extreme local MOEs of 10360 and 14200 N/mm² was 27%; the extreme deviations of the local MOEs vs. E_{glob} were –17% and 13.6%.

Table 3: *Compilation of local and global MOEs of test set B*

test	measured local MOEs in segments 1 to 6						global measured MOE	calculated MOE based on the measured local MOEs	location of failure
	E_{seg1}	E_{seg2}	E_{seg3}	E_{seg4}	E_{seg5}	E_{seg6}	E_{glob}	$E_{glob,calc}$	segment ¹⁾
B1	13555	14140	13375	13746	13746	14138	13779	13713	3
B2	15395	13687	12597	14032	14586	14212	14288	13703	2 - 3
B3	13808	11696	10356	13435	14203	12911	12508	12222	2 - 3
B4	17844	17843	16270	15825	15842	15366	16830	16333	4

¹⁾ x - y means the intersection between two segments

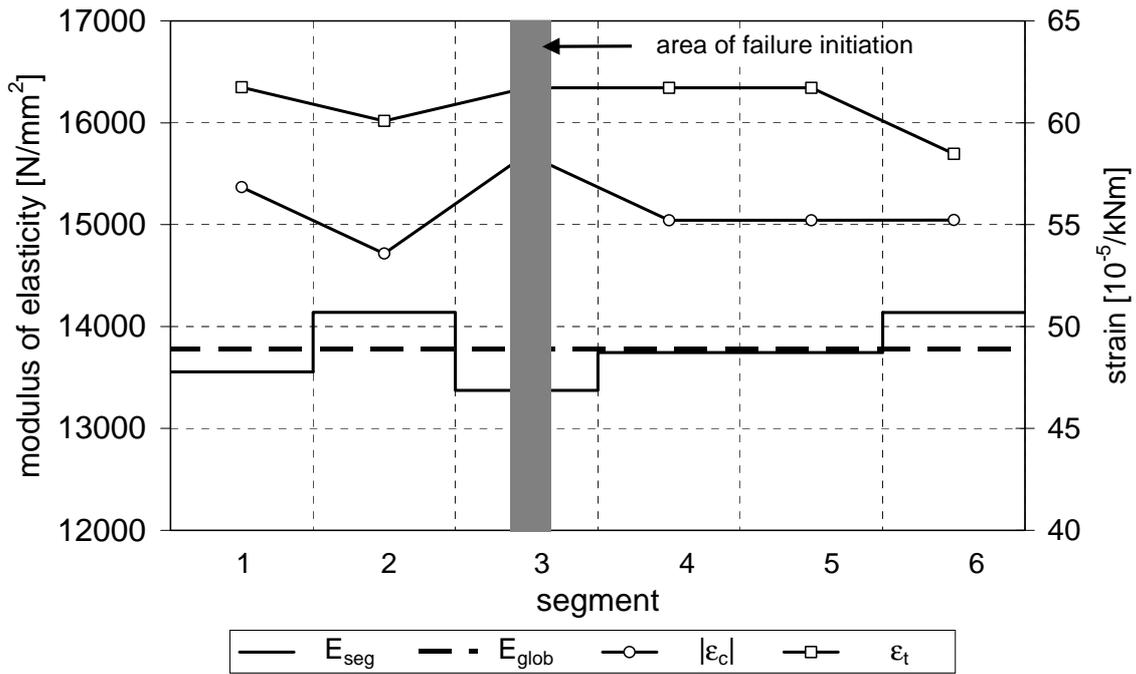


Fig. 6: Local strains, local and global MOEs of test B1 (board No. 416)

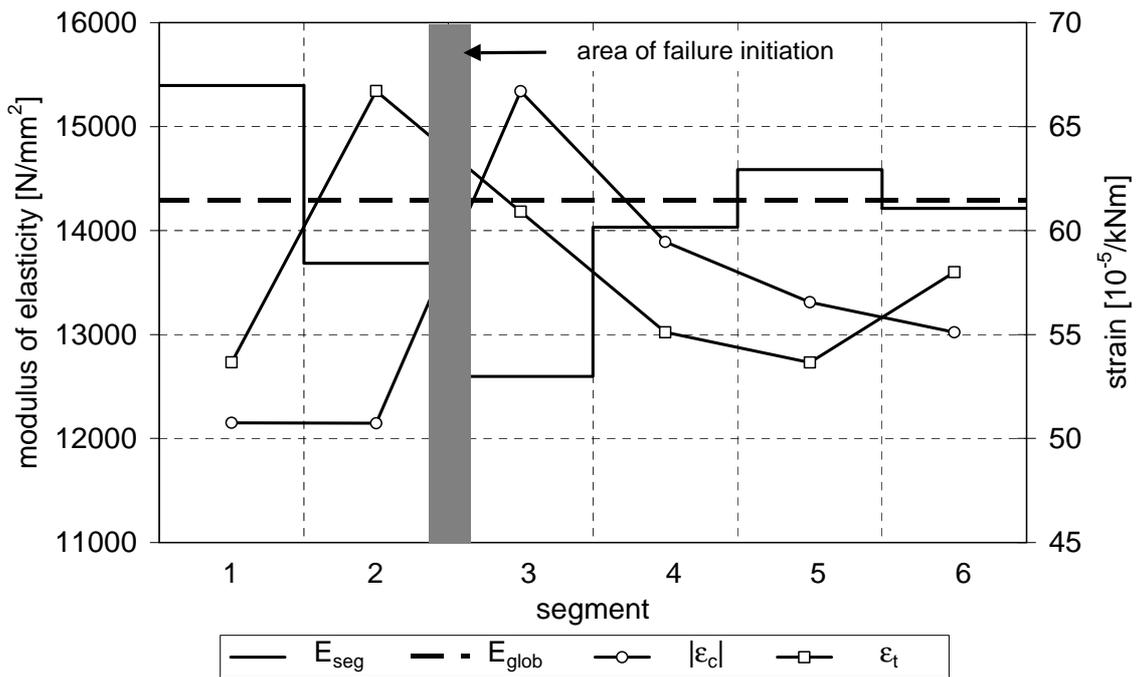


Fig. 7: Local strains, local and global MOEs of test B2 (board No. 258)

The specimen failed as sole specimen in the tests so far in bending compression at the transition of segments 2 and 3 with highest compression strains and lowest local MOE, respectively.

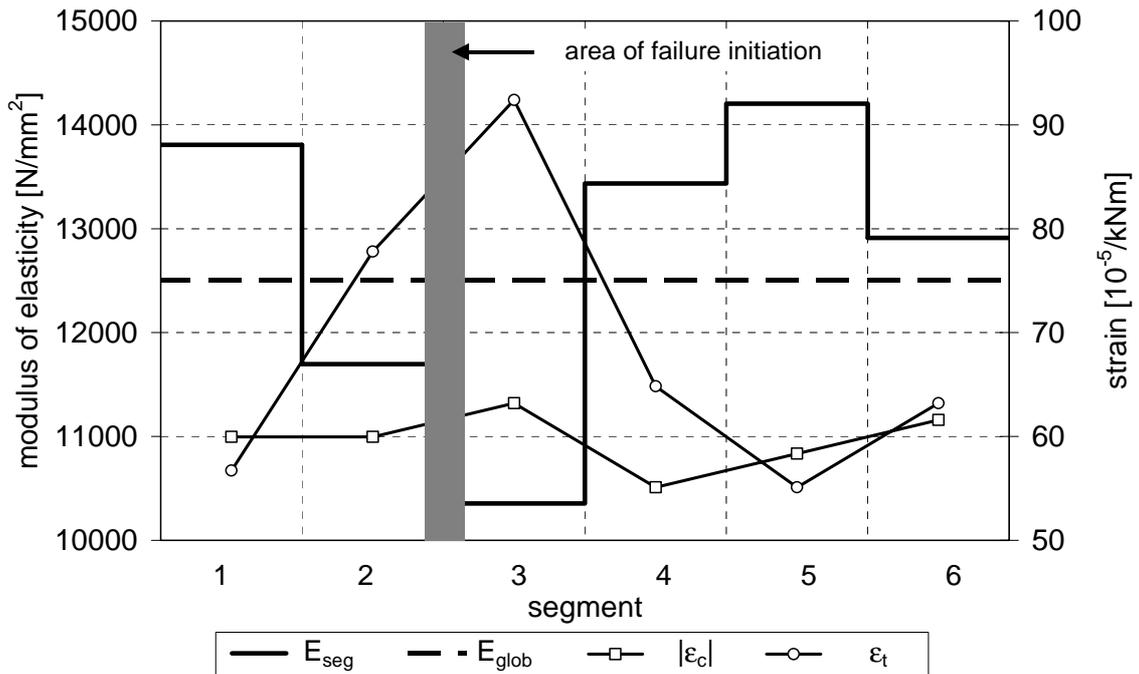


Fig. 8: Local strains, local and global MOEs of test B3 (board No. 281)

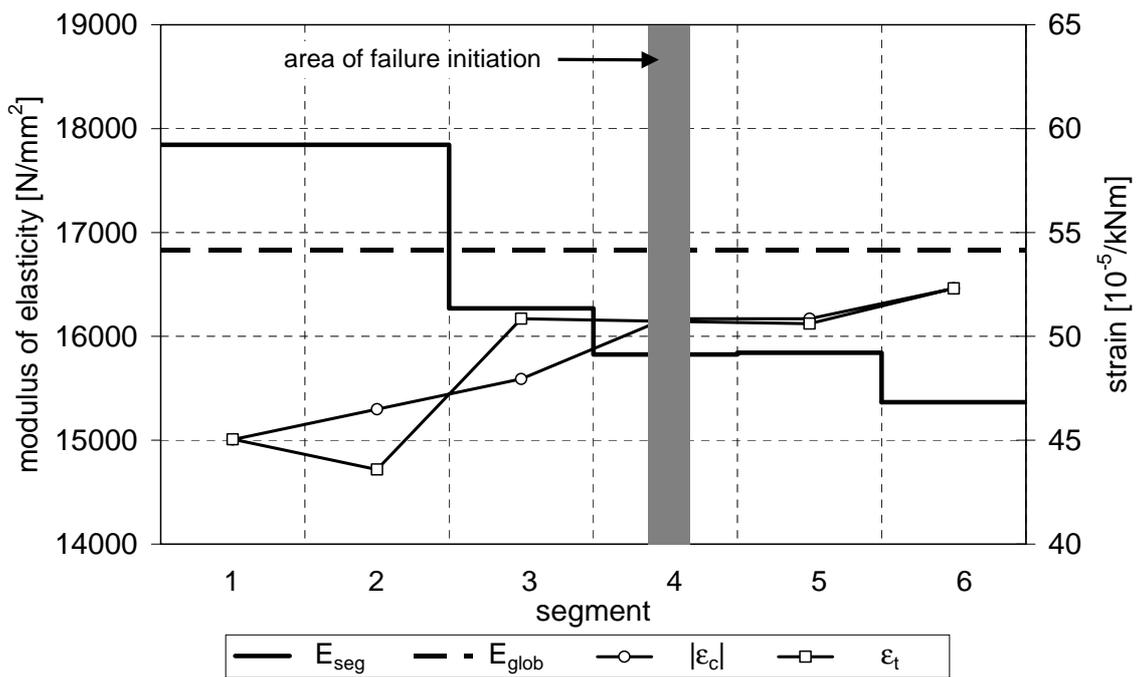


Fig. 9: Local strains, local and global MOEs of test B4 (board No. 213)

Figure 9 shows strains and MOEs of a board without knots and absolutely very “high” MOEs with a global MOE of 16830 N/mm². Bending compression and tension strains are very similar. The specimen failed in bending tension in segment 4 with the second lowest MOE. However, local MOEs in segments 4, 5 and 6 are very similar and deviate maximally by 2% from their respective mean.

Again, as in test set A, a very good agreement between the experimentally and computationally obtained global MOEs was observed. The deviation was in average 2.4% and maximally 3.7%.

6 DISCUSSION

The results show that the employed method is able to deliver local MOEs and therefore to reveal the MOE variation within a board. However, the measured local MOEs still do not represent the true MOEs of the zones with or without defects within the board. The measured MOE depends to a great extent on the gauge (segment) length, L , the length of the weak area and also on the relative differences of the stiffness within the gauge length. The smaller the chosen segment length, the smaller the difference between the measured and the “true” MOE. The employed gauge length of about 1.5 times the board depth seems to be in the size range of typical defect zones of the regarded wood species as the results show a good correlation between the minimum localized MOE value and location of bending failure. However, some improvement should still be obtained by a further reduction of gauge length L .

7 CONCLUSIONS

The presented results show that determination of the local modulus of elasticity in (edgewise) bending can be well performed by elongation/strain measurement at the bending tension and compression edges.

The measured local MOEs and the experimental global MOE obtained from deflection measurement, are consistent. This results from the fact that the global MOE can be predicted by beam theory or FE analysis with an average error of about 2% on the basis of the local MOE of the segments, here chosen with a length of 200 mm.

It was revealed that the locations of failure comply well with the locations of minimum MOE along beam length (the study so far comprised 30 beech boards). The presented method seems to enable the prediction of the type of bending failure either at the tension or compression edge.

The data of the on-going study serve as a calibration basis for modelling of the variation of modulus of elasticity and bending strength along beech wood boards as input data for glued compound elements with edgewise bent lamellas.

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