# APPLICATION OF THE MOMENT TENSOR INVERSION IN MATERIAL TESTING

# DIE ANWENDUNG DER MOMENTENTENSORINVERSION IN DER MATERIALPRÜFUNG

# APPLICATION DE L'INVERSION DU TENSEUR DES MOMENTS À L'ESSAI DES MATÉRIAUX

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## SUMMARY

Quantitative analysis of acoustic signals emitted during fracture processes within a body under load can be used to obtain valuable information about these processes directly. In addition to an accurate localization of the fracturing, an inversion for the moment tensor can be performed under certain assumptions, which yields information about the total energy released and the orientation of the rupture plane. After decomposition of the moment tensor, the type of rupture process can be quantified and visualized using ostensive crack models like those for shear and opening. In this article a brief introduction to moment tensor theory is given, some results of inverted moment tensors from a pull-out test are shown and various constraints are discussed.

## ZUSAMMENFASSUNG

Durch die quantitative Analyse von Schallsignalen, die von Bruchprozessen innerhalb eines unter Last stehenden Bauteils emittiert werden, können direkt zahlreiche Informationen über den Schädigungsverlauf gewonnen werden. Neben einer genauen Lokalisierung kann unter bestimmten Voraussetzungen auf den Momententensor invertiert werden, der Informationen über die Bruchenergie und die Orientierung der Bruchfläche beinhaltet. Nach geeigneter Zerlegung des Momententensors kann der Bruchmechanismus quantitativ und anschaulich durch verschiedene enthaltene Bruchmoden beschrieben werden. In diesem Artikel wird eine kurze Einführung in die Theorie des Momententensors gegeben, werden Ergebnisse invertierter Momententensoren eines Auszugversuches vorgestellt und verschiedene Einschränkungen der Methode diskutiert.

## RESUME

L'analyse quantitative des signaux ultrasoniques émis par les processus de rupture dans une structure sous chargement permet de recueillir directement de nombreuses informations sur le cours de la rupture. Outre une localisation précise, on peut dans certaines conditions obtenir par inversion le tenseur des moments, qui contient des informations sur l'énergie de rupture et l'orientation de la surface de rupture. Après la décomposition appropriée du tenseur des moments, le mécanisme de rupture peut être décrit quantitativement et visualisé par les différents modes de rupture impliqués. Cet article donne une brève introduction à la théorie des tenseurs des moments, présente les résultats de l'inversion des tenseurs des moments d'un essai d'arrachement et discute diverses restrictions de la méthode.


KEYWORDS: acoustic emission, moment tensor, fracture mechanism, concrete

## INTRODUCTION

Moment tensor inversion is used by seismologists to evaluate physical parameters of fault processes from seismic wave signals. Some of these parameters are the energy released during rupture, the orientation and the size of the rupture plane and the different fracture modes besides the classical double couple shear crack. This fracture model is sufficient to explain simple earthquake mechanics. For earth scientists who seek more information about petrological and mineralogical processes or for the supervision of the Nuclear Test Ban Treaty, however, a more detailed analysis of seismic events is necessary [DAHLEN & TROMP 1998].

For some decades, qualitative acoustic emission techniques have been widely used in material sciences and testing. The rapid development of measuring devices and computers over the last few years have allowed for the next step, which is the quantitative analysis of acoustic emissions. One of the goals of the non-destructive workgroup at the IWB is the evaluation of damage and fracture processes within concrete bodies using moment tensor inversion. The direct transfer of seismological methods to the material sciences may contain various uncertainties due to assumptions that are made for a simplification of the algorithms, such as neglecting near field terms or the model of a point source. Before moment tensor inversion can be applied as a standard method in material testing, these constraints and their effects have to be investigated systematically. Various studies were performed on this topic yielding promising results (e. g. [GROSSE 1996], [OHTSU et al. 1991]).

## RADIATION PATTERNS OF VARIOUS FRACTURE TYPES

The classical model for an earthquake fracture is the pure shear crack along a planar rupture surface. In Figure 1 the radiation pattern for this type of fracture is illustrated for the compressional (P-) and the shear (S-) wave, in grey – dark for an expanding (+) incident particle motion and light a contracting (-) motion – and white respectively. The coils represent the spatial amplitude or energy distribution and the polarities. Along the nodal planes there is no particle motion. To distinguish which of the two nodal planes is the rupture plane, the polarities of the shear wave with an oscillation perpendicular to the direction of the wave propagation or additional external information (e. g. the stress regime) have to be taken into account.

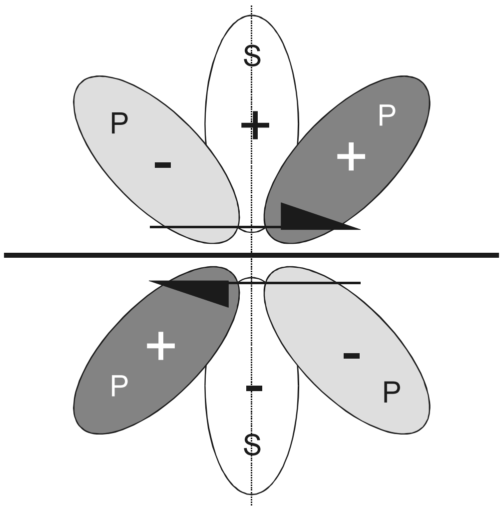


*Figure 1: Radiation patterns of a dextral shear crack for the compressional (P-) wave and the horizontal polarized shear (S-) wave.*

With only minimal information from the transient waveforms of an event from several seismological stations, which are distributed on the earth's surface, focal solutions can be calculated [FOWLER 1990].

However, this simple model of a shear crack can not describe more complex rupture phenomena, where opening of a crack and isotropic components are present. In Figure 2 three different rupture modes with their radiation patterns are shown. As a first assumption the source of the acoustic emission is viewed as a point source from which seismic energy is radiated into the specimen as an elastic wave. To illustrate the particle motion in space, a half sphere is situated at the source (see the circle) and the amplitudes (in grey scales) and polarities (in black and white) are projected onto it and plotted in a stereographic projection.

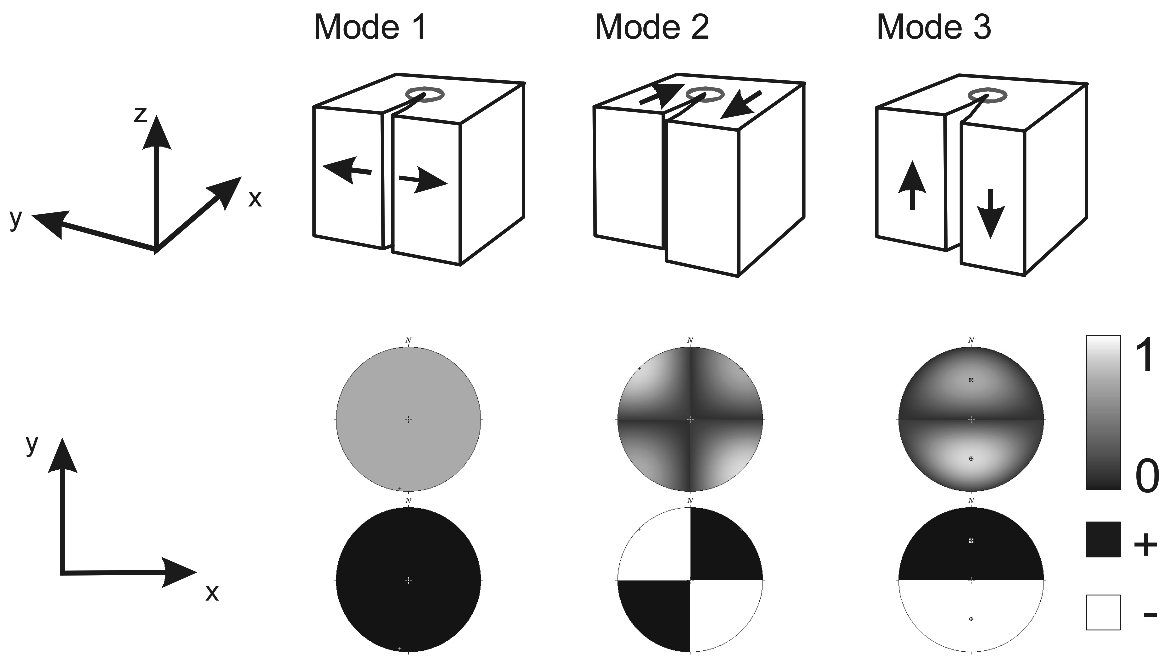


*Figure 2: Comparison of radiation patterns of an opening crack (mode 1) and shear cracks (mode 2 and 3). On top an illustration of the rupture zone, in the middle the distribution of the amplitudes in a grayscale and on bottom the incident polarities of the focal particle motion, both in a stereographic projection.*

For the mode 1 crack, the incident particle motion is radial outwards, similar to an explosion. In the ideal case the energy distribution is equal in all directions. The collapse of a pore could be explained with a mode 1 event with negative sign, which means an incident particle motion towards the hypocenter. Mode 2 and mode 3 both are shear cracks with a progress of the tip of the crack parallel or perpendicular to the movement of the 'sliding blocks' respectively. For the first, the rupture plane would be in the x-direction with the rear block moving to the right which means an outwards particle motion on the right and an inwards on the left. For the latter, the rear block would be lifted relative to the front. In material sciences most cracks can not be described by only one of these simple source types. Mixtures of the three modes above and other phenomena are necessary to explain failure processes.

## A DISCRIPTIVE PRESENTATION OF THE MOMENT TENSOR

In the early years of seismological studies, the strength of an earthquake was often ‘measured’ in subjective scales by an evaluation of visible damages (e. g. Mercalli’s intensity). Later, ground motions at seismographic stations were transformed into a magnitude (e. g. Richter’s scale), but the comparability between different observations was not satisfactory. Interest in a detailed description of the spatial distribution of energy in relation to the source led to the moment tensor. This tensor contains information about the total energy, the orientation of the nodal planes with respect to the shear crack component and the isotropic component. A suitable decomposition of the moment tensor yields various features of the rupture process.

The moment tensor $M_{ij}$ with the unit of a moment of force is a symmetric 3x3-tensor containing 6 independent elements:

$$M_{ij} = \begin{pmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{pmatrix}.$$

The physical meaning of the various elements is illustrated in Figure 3.

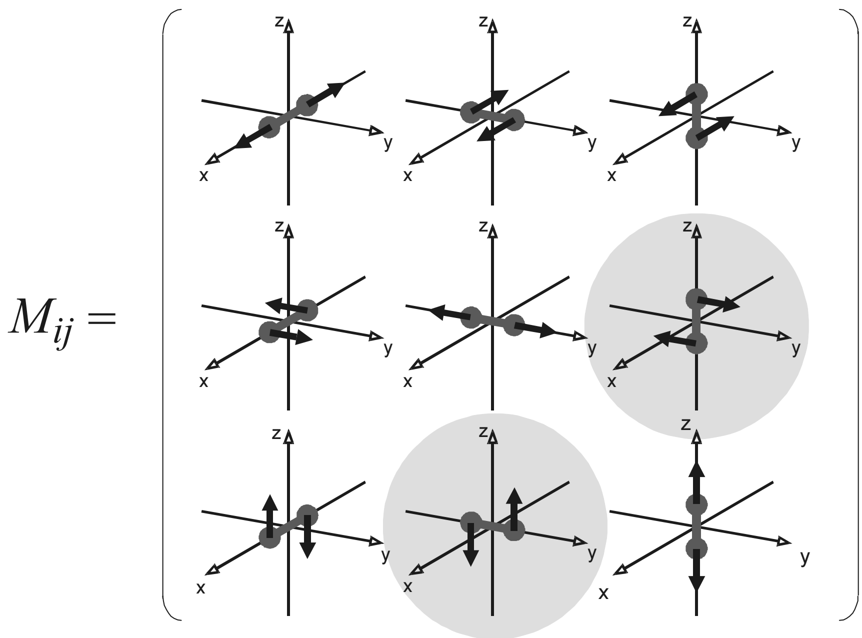


*Figure 3: The elements of the moment tensor as dipoles in the spatial dimensions and the acting forces, which lead to moments of force.*

The components of the moment tensor can be thought of dipoles (see the joints) oriented in the three spatial dimensions (columns of $M_{ij}$) on which ends forces (see the arrows) act in the three spatial dimensions (rows of $M_{ij}$). On the diagonal the axes of the joints are parallel to the forces, and describe the volumetric change. For the other elements, the forces lead to a torque around the axis perpendicular to the plane containing the force and the dipole. To obtain a total zero moment of forces, the model of the double couple is useful to show the symmetry of the tensor (see Figure 4) – for example for the two marked elements $M_{yz}$ and $M_{zy}$.

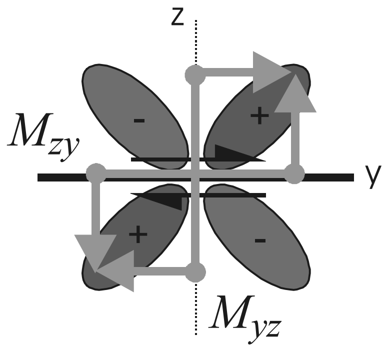


*Figure 4: The radiation pattern of a pure shear event with the corresponding pair of double couples. The symmetry of the tensor yields a zero total torsion.*

The two elements $M_{yz}$ and $M_{zy}$ with

$$M_{yz} + M_{zy} = 0$$

build a double couple, which can lead to a pure shear crack when the stress in the material exceeds a critical mechanic threshold. Dependent on the global stress regime or the constitution of the material, a crack will occur in the *xy*-plane or the *xz*-plane and the stress is released by a relative slip of the upper part to the right or a relative slip of the right part upwards respectively. Again the radiation pattern of such an event is illustrated.

The decomposition of the moment tensor $\underline{M} = M_{ij}$ with

$$\underline{M} = \underline{M}^{ISO} + \underline{M}^{DC} + \underline{M}^{CLVD}$$

can be performed as was done by JOST & HERMANN [1989] into an isotropic part $\underline{M}^{ISO}$ describing a volumetric change and two deviatoric parts, a shear crack following the double couple mechanism $\underline{M}^{DC}$ and a compensated linear vector dipole $\underline{M}^{CLVD}$ (see Figure 5).

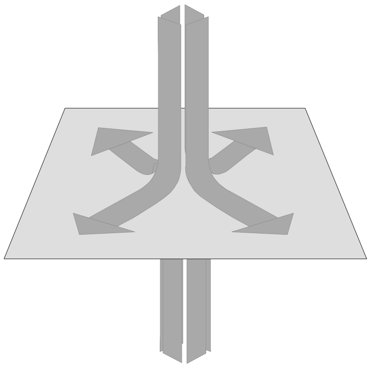

*Figure 5: Principle of the model of the compensated linear vector dipole. A compressional particle motion in one plane is compensated by a dilatational motion normal to it – or vice versa. The change in volume is zero.*

The intensity of the event, or its relative Moment $M_r$, is given by the sum of the deviatoric components. The energy of the isotropic component is given in relation to $M_r$ with a positive sign for expansion.

## THE INVERSION OF THE MOMENT TENSOR

The forward problem of the propagation of elastic waves in a brittle medium is formulated in detail in AKI & RICHARDS [1980]. Typically a certain source function generates an elastic wave that is propagating through a medium and is affected by attenuation, dispersion, reflection, refraction and more. To measure these waves different types of sensors can be attached on the surface of the specimen. Every sensor itself has a certain transfer function yielding a filtered electric signal which can be recorded. In Figure 6 this procedure is illustrated.

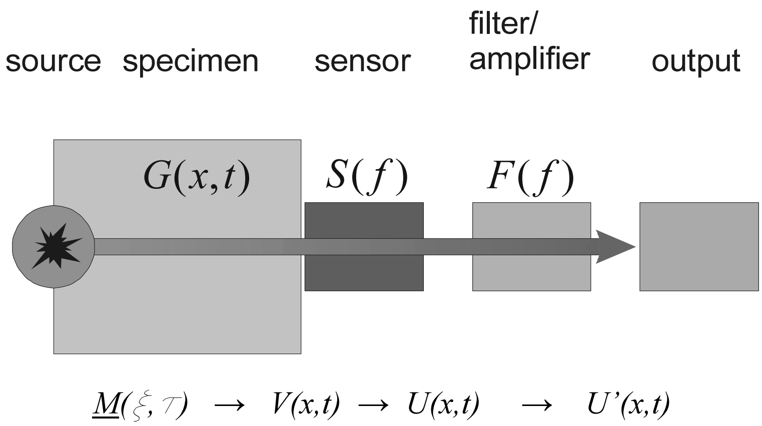


*Figure 6: The forward problem of the propagation of an elastic wave as a convolution of the source function and various transfer functions.*

To gain information about the source, the recorded data have to be inverted after the filtering effects of the medium and the sensors etc. are removed by a deconvolution. While determination of the transfer functions of the measuring equipment is possible, this is very difficult for the Greens' functions *G(x,t)* including all the effects on elastic waves in the medium. A relative moment tensor inversion introduced by DAHM [1993] solves this problem. For a number of different events originating out of a relatively small spatial and temporal cluster, the travel paths to the sensors are nearly the same and so are the effects on these waves by the medium. Under several further assumptions, like the validity of the ray theory for elastic waves, a point source and a homogenous and isotropic medium, as well as the neglection of near field terms, the source functions of the events in the cluster can then be calculated from their relative amplitude distributions. Therefore the amplitudes of the first P-wave peaks in the recorded transient waveforms have to picked. The justification of these assumptions for an application of the relative moment tensor inversion in the material sciences is a major topic of the nondestructive workgroup at the IWB (e. g. GROSSE [1996], WEILER [2000]).

## EXAMPLE OF EXPERIMENTAL INVESTIGATIONS

Within the scope of the collaborative research center SFB 381 a pull-out-test of a steel bar (Ø 16 mm) with five ribs out of a 20 cm concrete cube was realized. The acoustic emissions were recorded by an 8-channel transient recorder with a sample rate of 1 MHz per channel. Eight piezo-electric accelerometers were evenly distributed on the surfaces of the cube. In total we recorded some 230 acoustic emissions. The total number of events was much higher, but due to the delay of the recorder during the readout of every single event on the hard drive, only a small number of them could be recorded.

On the left side of Figure 7 a sketch of the cube and the bar is shown. On the right side the locations of acoustic emissions are illustrated, the black spots mark the cluster discussed later. As expected, most of the events occur in the central area of bond, but there are some single events along the stress-trajectories. Due to stress between the non planar surface of the specimen and the support, several emissions show up in the right of the picture. The error in the localization is about some 0,5 cm.

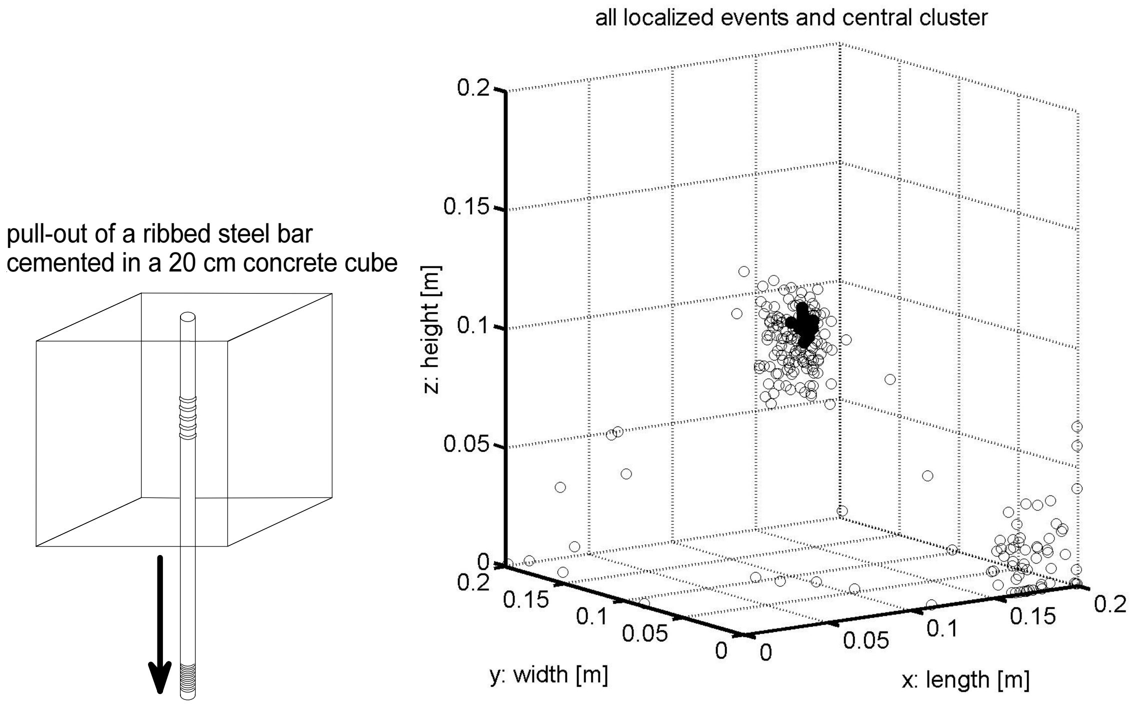


*Figure 7: Left: Sketch of the specimen, a 20 cm concrete cube with a ribbed steel bar (Ø 16 mm) cemented in its central axis. Right: The locations of the acoustic emissions due to the pull-out in the central area. The black dots indicate the cluster investigated later. The events in the right corner are related to stress in the no planar surface of the specimen on the support.*

## Quantitative analysis of the acoustic emissions

Because of their localization, a cluster of 14 events in the central area of the specimen was investigated in more detail (see Figure 8). The geometry of the test suggests opening cracks, as well as shear cracks, with vertical rupture planes, more or less concentric around the axis of the steel bar.

The amplitudes were picked first out of the raw data. Out of the cluster of 14 events several sub-clusters were composed to study the requirements for the temporal and spatial clustering and the stability of the solutions. The single acoustic emissions were therefore investigated in different groups and inverted for their moment tensors.

Typically solutions of the moment tensor inversions for two selected events are illustrated in Figure 8. In addition to the radiation pattern of seismic energy, Figure 8 shows the relative seismic moment $M_r$ with error and the percentile distributions of isotropic (*ISO*) and double-couple (*DC*) source mechanisms. For the latter, the solution of the inversion is given in front of the brackets, in the brackets there is the solution of a boot-strap-analysis (BSA) [e. g. EFRON &

TIBSHIRANI 1986] with its error. The more similar the results of the BSA and the inversion and the smaller the errors, the better the solution is.

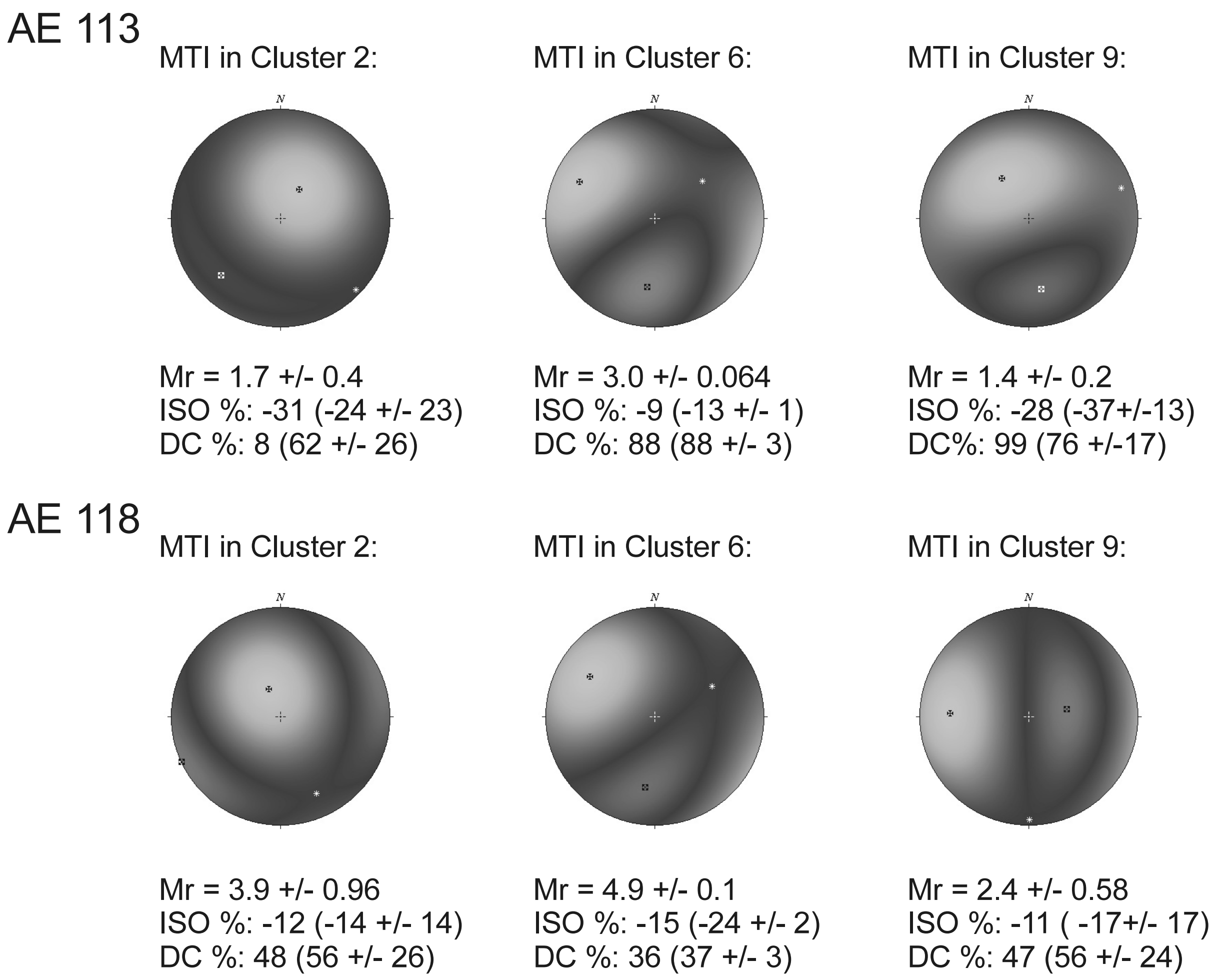


*Figure 8: Solutions for the relative moment tensor inversion of events 113 and 118. Illustrated is the distribution of the focal amplitudes of the events in a grayscale. The inversion was performed in three subclusters, on the right (Cluster 9) all events were involved.* $M_r$ *is the relative seismic moment or the intensity of the emission.*

It is inconvenient that the isotropic component has a negative sign, which would suggest an implosive character. Additionally there are no unique solutions for both events. Event 118 has a stable ratio between *ISO* and *DC* components, but the orientation of the nodal planes varies and the errors are very high. For event 113 the inversion in cluster 2 does not fit to those in cluster 6 and 9 at all and has also high errors. For the inversions in cluster 6 and 9 the orientations of the nodal planes is relatively stable. Both events show a significant isotropic component, but the shear components are prevailing.

The number of events in the investigated clusters is relatively small and the focal mechanisms probably too similar, which makes the analysis of the data with the discussed relative inversion method vulnerable to noise in the data (compare DAHM [1993]). With a faster transient recorder more events could be acquired and these effects would be reduced.

The transfer functions of the sensors are dependent on frequency. The frequency content of the signals varies with size and type of the source and it is not obvious, if the transfer functions of the sensors can be totally eliminated from the calculations by the relative inversion. By a calibration of the acquisition system and a deconvolution of its transfer function this could be avoided.

The good localization of the events is an indicator that the tested specimen was sufficiently homogenous and isotropic. The processed signals have wavelengths in the range of a few centimetres. If it has to be assumed that the near field terms of the sources still contribute energy to the signals, then a modification of the moment tensor inversion algorithms would be necessary.

## CONCLUSIONS

Inversion of moment tensors is a standard procedure in seismological investigations and yields the orientation of the rupture plain and the seismic energy information about the crack type. The application of this method to the material sciences is promising. A good localization of crack events in concrete due to its sufficient homogeneity and isotropy can be realized. For high quality data, this can be performed semi- and full automatic with the *WinPecker* program [GROSSE 2000]. The 'know-how' gathered at the IWB on moment tensor inversion allows for an experienced application of the procedures and steps have been made to develop more user-friendly, automatic routines. There is still need for a more detailed investigation of theoretical aspects.

Significant improvement of the stability of the solutions is expected from the calibration of the sensors.

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