LINEAR VERSUS QUADRATIC FAILURE CRITERIA FOR INPLANE LOADED WOOD BASED PANELS

EINE GEGENÜBERSTELLUNG LINEARER UND QUADRATISCHER VERSAGENSKRITERIEN FÜR SCHEIBENBEANSPRUCHTE HOLZ-WERKSTOFFPLATTEN

UNE COMPARAISON DES CRITERES DE RUPTURE LINEAIRE ET QUADRATIQUE POUR DES PANNEAUX DE LAMELLES ORIENTEES SOLLICITES DANS LE PLAN

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SUMMARY

The paper deals with different failure criteria applied to off-axis tension strength of in-plane loaded oriented strand board (OSB) panels. The four criteria regarded are: a linear and a quadratic approach, the Tsai-Wu tensor polynom and a criterion based on the von Mises invariant.

Application to an extensive experimental data set of OSB boards of different thicknesses shows a clear superiority of the von Mises invariant criterion which delivers an almost ideal fit of the experimental data, both, on the mean and the characteristic strength level. Contrary hereto, the linear criterion gives a far too conservative strength prediction.

It is proposed to replace in the present draft of the German timber design code the off-axis strength equation for OSB boards based on the linear criterion by the strength equation based on the von Mises invariant.

ZUSAMMENFASSUNG

Der Aufsatz beschäftigt sich mit unterschiedlichen Versagenshypothesen zur Beschreibung der “off-axis” Zugfestigkeit von scheibenbeanspruchten OSB-Flachpreßplatten. Die vier betrachteten Hypothesen umfassen einen linearen und einen quadratischen Ansatz, das Tsai-Wu Tensorpolynom und eine Hypothese, die auf der von Mises Invariante fußt.

Die Anwendung auf einen umfangreichen experimentellen Datensatz für OSB-Platten unterschiedlicher Dicken zeigt eine klare Überlegenheit der von Mises Invarianten-Hypothese auf. Dieser Ansatz erbringt eine nahezu ideale

Anpassung der experimentellen Ergebnisse sowohl auf dem Mittelwertsniveau als auch auf dem 5%-Fraktilenlevel. Im Gegensatz hierzu ergibt die lineare Versagenshypothese deutlich zu konservative Festigkeitswerte.

Es wird vorgeschlagen, die “off-axis” Festigkeitsgleichung für OSB-Platten zufolge des linearen Kriteriums im jetzigen Entwurf der deutschen Holzbaunorm durch die Festigkeitsgleichung basierend auf der von Mises Invariante zu ersetzen.

RESUME

Dans cet article sont analysés différents critères de rupture en traction hors axe pour des panneaux de lamelles orientées (OSB) sollicités dans le plan. Les 4 critères étudiés sont : deux critères basés sur une approche linéaire ou quadratique, le critère polynomial de Tsai-Wu et un critère de type von Mises.

L’application à une base de données extensive de panneaux d’OSB d’épaisseurs diverses fait clairement apparaître la supériorité du critère invariant de von Mises, qui ajuste de manière quasi-parfaite les données expérimentales, aussi bien en moyenne que sur les valeurs caractéristiques. Par opposition, le critère linéaire se révèle beaucoup trop conservatif.

On propose de remplacer le projet de code allemand de construction bois l’équation de résistance hors axes de panneaux d’OSB basée sur le critère linéaire par une équation basée sur le critère de von Mises.

KEYWORDS: linear and quadratic failure criteria, von Mises invariant criterion, oriented strand board (OSB), off-axis design strength equation

1. INTRODUCTION

The draft of the new German timber design code gives a presently not specified off-axis strength equation for in-plane tension loaded wood-based panels such as OSB and plywood. The equation is based on a linear interaction of the stress/strength ratios of the two normal stresses and of the shear stress. It represents an extension of the Hankinson equation, specified in many design codes for solid wood and glulam, by the shear term. The quality of the proposed approach is not widely known apart from the derivation inherent fact that it is a very conservative solution.

This paper aims at a quantitative comparison of some strength criteria for prediction of off-axis tension strength of oriented strand board (OSB) panels. In detail, the linear criterion of the draft proposal is compared to three criteria of quadratic nature. Apart from the basic quadratic criterion, the Tsai-Wu tensor polynom approach and a criterion based on the von Mises invariant are regarded. The latter criterion is not known for application to structural wood-based materials so far.

2. LINEAR FAILURE CRITERION

The simplest perceivable strength interaction criterion for a biaxial stress state in orthotropic materials is linear; so, in case of in-plane biaxial on-axis loading (on-axis = parallel to axes x, y of material orthotropy) the linear failure criterion is

$$\frac{\sigma_x}{f_x} + \frac{\sigma_y}{f_y} + \frac{\tau_{xy}}{f_v} = 1 \tag{1}$$

where f_x and f_y are on-axis normal strengths (compression or tension) and f_v is on-axis shear strength. For normal tension stresses, here regarded exclusively, the strength notation is $f_x = f_{x,t}$, $f_y = f_{y,t}$.

For a deliberate off-axis stress state in the 1, 2 co-ordinate system, rotated by angle α against the on-axis system, on-axis stresses σ_x, σ_y, τ_{xy} are replaced by the transformed stresses σ_1, σ_2, τ_{12} according to the usual transformation equation

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \tag{2}$$

where m = cos α, n = sin α.

So, in case of uniaxial off-axis loading with σ_1 at angle α to on-axis direction we have

$$\sigma_x = \cos^2\alpha \, \sigma_1,$$

$$\sigma_y = \sin^2\alpha \, \sigma_1,$$

$$|\tau_{xy}| = \cos\alpha \, \sin\alpha \, \sigma_1 \tag{3a-c}$$

and the linear interaction criterion (1) reads

$$\sigma_1 \left(\frac{\cos^2\alpha}{f_{x,t}} + \frac{\sin^2\alpha}{f_{y,t}} + \frac{\cos\alpha \, \sin\alpha}{f_v} \right) = 1 \quad . \tag{4}$$

Using for the failure state the notation $\sigma_1 = f_{\alpha,t}$, one obtains

$$f_{\alpha,t} = \frac{f_{x,t}}{\frac{f_{x,t}}{f_{y,t}} \sin^2\alpha + \cos^2\alpha + \frac{f_{x,t}}{f_v} \cos\alpha \sin\alpha} \quad . \tag{5}$$

Equation (5) is the off-axis tension (and bending) strength expression given in the draft of the German timber design code E DIN 1052 (N.N. 2000a) for wood-based materials such as oriented strand board (OSB) and plywood.

If the shear term in eq. (1) is omitted, the denominator in eq. (5) reduces to the first two terms. The resulting equation

$$f_{\alpha,t} = \frac{f_{x,t}}{\frac{f_{x,t}}{f_{y,t}} \sin^2\alpha + \cos^2\alpha} \tag{6}$$

is then the so-called Hankinson formula which is employed in Eurocode 5 and in E DIN 1052 for off-axis tension loading of solid wood, glulam and uniaxially oriented laminated veneer lumber (LVL). It should be stated that the failure criterion acc. to Hankinson is actually based on the first invariant of the plane stress tensor, namely the sum of the normal stresses.

The reason for the neglect resp. for the recognition of the shear term in the linear interaction equation (1) for wood and wood-based materials is based on the assumption of its different relevance for the respective materials. For wood and glulam the linear interaction with omittance of the shear term proofed to be a reasonable good approximation for off-axis tension and compression strength. The applicability/accuracy of eq. (5) for wood-based materials has not been revealed publically so far.

3. QUADRATIC FAILURE CRITERION

The most simple quadratic strength criterion resulting in an ellipsoidal strength envelope, can be written as

$$\left(\frac{\sigma_x}{f_x}\right)^2 + \left(\frac{\sigma_y}{f_y}\right)^2 + \left(\frac{\tau_{xy}}{f_v}\right)^2 = 1 \quad . \tag{7}$$

Replacing the on-axis stresses by off-axis stresses (eqs. 3a-c) and using the notation $\sigma_1 = f_{\alpha,t}$, we obtain

$$f_{\alpha,t} = \frac{1}{\sqrt{\dfrac{\cos^4\alpha}{f_{x,t}^2} + \dfrac{\sin^4\alpha}{f_{y,t}^2} + \dfrac{\cos^2\alpha\sin^2\alpha}{f_v^2}}} \quad . \tag{8}$$

It will be shown that the quadratic strength criterion (7) represents a special case of the Tsai-Wu criterion discussed below.

An important general difference comparing the quadratic failure criterion with the linear criterion can be stated. Regarding the extreme values of the off-axis strength eqs. (5) and (8) a significant difference between both approaches can be easily verified by differentiation and solving for angle α. The linear strength criterion always shows a global minimum of strength $f_{\alpha,t}$ between $\alpha = 0°$ and $\alpha = 90°$. Contrary hereto, the extreme values of the quadratic criterion may also be located at the boundaries of the angle range what represents a strongly increased flexibility for approximation of experimental data; the latter becomes evident from below given data approximations.

4. TSAI-WU TENSOR POLYNOM CRITERION

Although a purely hypothetical approach, the second order tensor polynom by Tsai and Wu (1971), being invariant to co-ordinate transformation, provides a considerably more versatile tool for handling biaxial stress states. Contrary to the linear and quadratic criteria stated in paragraphs 2 and 3, it allows the occurance of mixed positive and negative signs of the normal stresses. Further, a coupling term between both normal strengths is included. Following, the relevant equations are developed in short manner; for further details it is referred to Tsai and Hahn (1980). The failure criterion in abbreviated index notation reads

$$F_{ij}\ \sigma_i\ \sigma_j + F_i\ \sigma_i = 1 \tag{9}$$

where the F terms are strength parameters and in the on-axis case the notation i, j = x, y, s is used.

Equation (9) can be expanded (with $F_{xs} = F_{ys} = F_s = 0$, resulting from insignificance of the sign of shear stress on strength) to

$$F_{xx}\,\sigma_x^2 + 2F_{xy}\,\sigma_x\,\sigma_y + F_{yy}\,\sigma_y^2 + F_{ss}\,\sigma_s^2 + F_x\,\sigma_x + F_y\,\sigma_y = 1. \tag{10}$$

There are four quadratic and two linear strength parameters in eq. (10) defined as

$$F_{xx} = \frac{1}{f_{x,t}\ \ f_{x,c}},\ F_{yy} = \frac{1}{f_{y,t}\ \ f_{y,c}},\ F_{ss} = \frac{1}{f_v^2},\ F_{xy} = \sqrt{F_{xx}\ \ F_{yy}}\left(-1/2\right) \tag{11a-d}$$

$$F_x = \frac{1}{f_{x,t}} - \frac{1}{f_{x,c}},\quad F_y = \frac{1}{f_{y,t}} - \frac{1}{f_{y,c}}\quad . \tag{12}$$

The normal stress interaction term F_{xy}, as specified in eq. (11d), is related to the assumption that eq. (9) is a generalized von Mises criterion; for details see Tsai and Hahn (1980).

The on-axis failure criterion (10) can be easily transformed into an off-axis loading condition by substituting the on-axis stresses σ_{ij} with i,j = x,y,s by off-axis stresses σ_{ij} with i,j = 1, 2, 6 through transformation relationship (2). For the regarded case of uniaxial off-axis (tension) loading with stress σ_1 acc. to eqs. (3a-c), the transformation results in the expression

$$F_{11}\,\sigma_1^2 + F_1\,\sigma_1\ = 1 \tag{13}$$

where

$$F_{11} = \cos^4\alpha\, F_{xx} + \sin^4\alpha\, F_{yy} + 2\cos^2\alpha \sin^2\alpha\; F_{xy} + \cos^2\alpha \sin^2\alpha\, F_{ss}\,,$$

$$F_1 = \cos^2\alpha\, F_x + \sin^2\alpha\, F_y \quad . \tag{14a,b}$$

Explicitly one can write for $\sigma_1 = f_{\alpha,t}$ (Note: off-axis tension strength = positive root)

$$\begin{aligned} f_{\alpha,t} = 2 / \Bigg\{ & \cos^2\alpha \left(\frac{1}{f_{x,t}} - \frac{1}{f_{x,c}}\right) + \sin^2\alpha \left(\frac{1}{f_{y,t}} - \frac{1}{f_{y,c}}\right) + \\ & + \Bigg[4\cos^2\alpha \left(\frac{\cos^2\alpha}{f_{x,t} f_{x,c}} + \sin^2\alpha \left(\frac{1}{f_v^2} - \sqrt{\frac{1}{f_{x,t} f_{y,t} f_{x,c} f_{y,c}}} \right)\right) + \\ & + \left(\cos^2\alpha \left(\frac{1}{f_{x,t}} - \frac{1}{f_{x,c}}\right) + \sin^2\alpha \left(\frac{1}{f_{y,t}} - \frac{1}{f_{y,c}}\right)\right)^2 + \frac{4\sin^4\alpha}{f_{y,t} f_{y,c}} \Bigg]^{1/2} \Bigg\} \end{aligned} \quad . \tag{15}$$

For the special case of equality of tension and compression strength in x- and y- directions and neglecting the interaction term (11d) of the normal stresses, F_{xy}, eq. (15) simplifies to the strength equation of the quadratic failure criterion stated in eq. (8). Hence, the quadratic failure criterion may be perceived as a special case of the Tsai-Wu tensor polynom approach.

5. VON MISES INVARIANT CRITERION

A promising approach for a failure criterion generally consists in the use of invariants of the stress tensor (see eq. (6)). Now, the von Mises invariant, as suggested by Tsai (1988) and Kim (1995), is regarded. We start from the von Mises yield equation, being an invariant of the stress tensor, which for plane stress condition reads

$$\sigma_v^2 = \sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2 \quad . \tag{16}$$

The strength criterion is obtained by normalization to respective strength components, giving for the on-axis case

$$\left(\frac{\sigma_x}{f_x}\right)^2 - \frac{\sigma_x \sigma_y}{f_x f_y} + \left(\frac{\sigma_y}{f_y}\right)^2 + 3\left(\frac{\tau_{xy}}{f_v}\right)^2 = 1 \quad . \tag{17}$$

It should be stated that eq. (17) is very close to a strength criterion given by Norris (1962); the difference consists in the factor 3 of the shear strength term. The Norris criterion enables a good approximation of some orthotropic fiber reinforced plastics; for timber it showed less good predictions compared with the linear Hankinson approach (6).

For the off-axis case we obtain by introducing eqs. (3a-c) and the notation $\sigma_1 = f_{\alpha,t}$

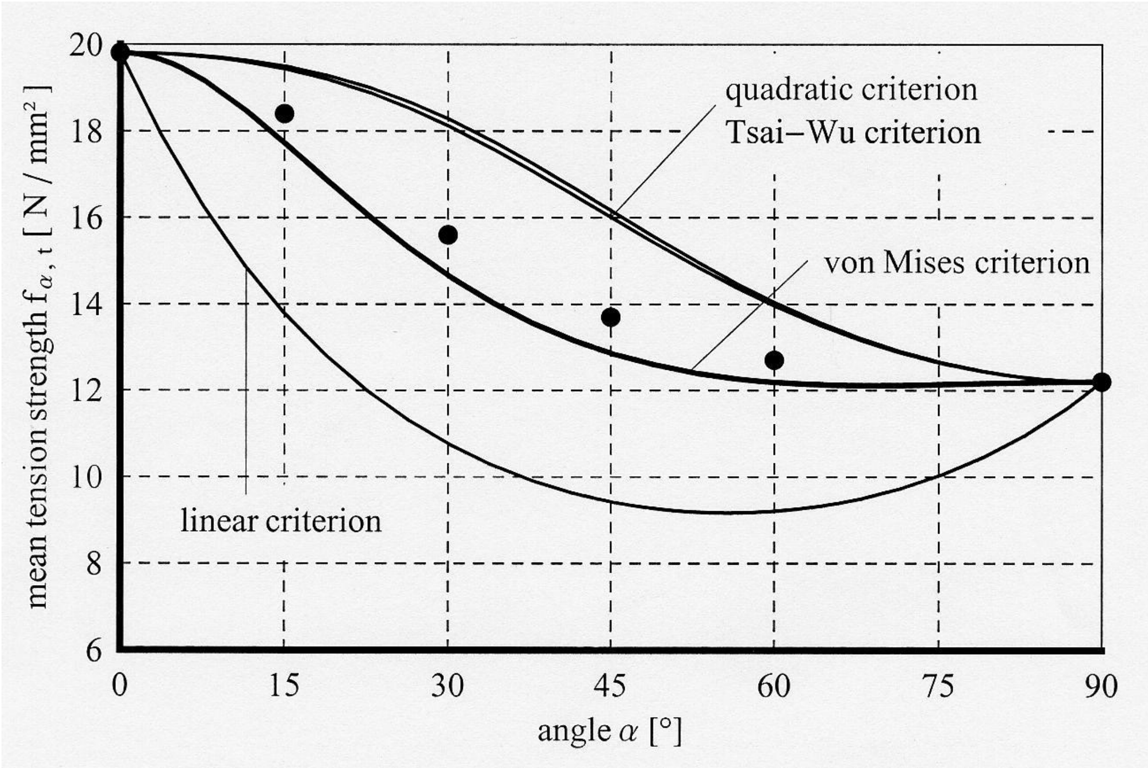
$$f_{\alpha,t} = \frac{1}{\sqrt{\frac{\cos^4\alpha}{f_{x,t}^2} + \frac{\sin^4\alpha}{f_{y,t}^2} + \left(\frac{3}{f_v^2} - \frac{1}{f_{x,t} f_{y,t}}\right)\cos^2\alpha\sin^2\alpha}} \quad . \tag{18}$$

6. APPLICATION TO EXPERIMENTAL DATA

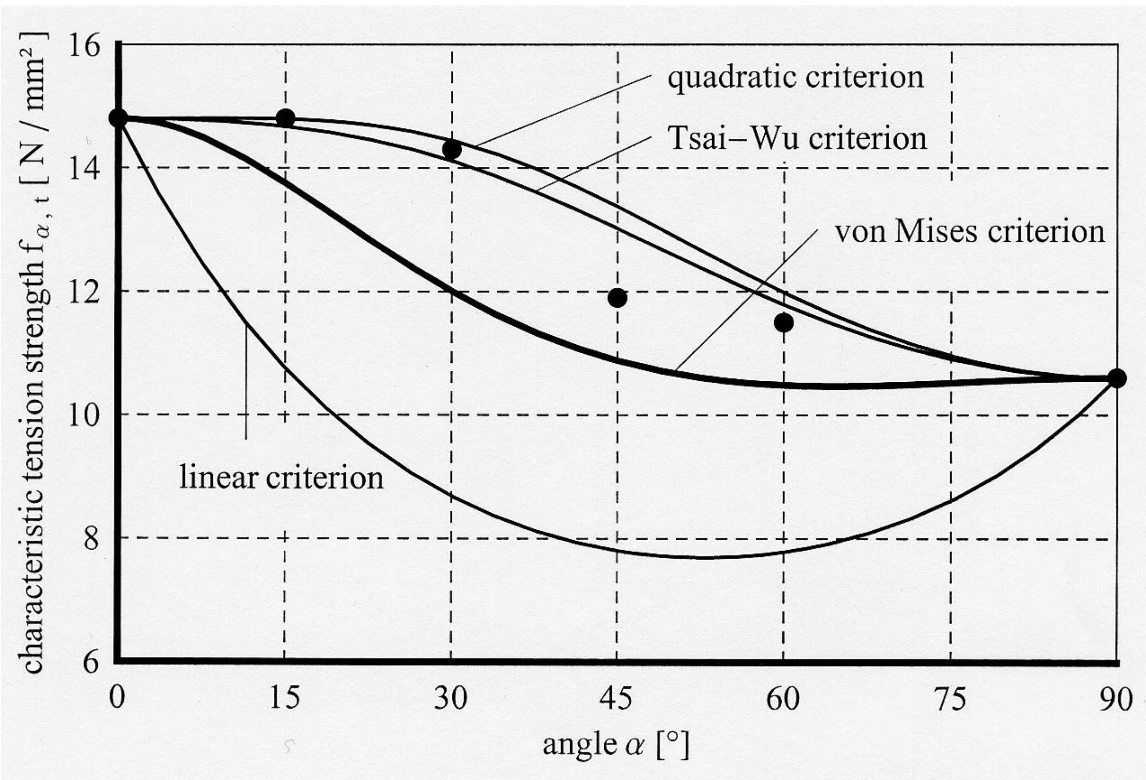
Following the aptness of the discussed strength criteria for approximation of off-axis tension strength of wood-based materials is demonstrated exemplarily for a set of oriented strand board (OSB) data. The extensive experimental data were obtained in tests for a German Technical approval (N.N. 2000b) performed at Otto-Graf-Institute. Table 1 specifies the on-axis strength data on the mean and 5 % fractile level for panels of three different thicknesses of 10, 15 and 18 mm.

Table 1: Compilation of mean and 5% fractile values of on-axis strength data obtained in technical approval testing for a specific OSB/4 board type (N.N. 2000b) for different panel thicknesses

Strength components		On-axis strengths in N/mm² of OSB/4 boards of different thicknesses					
		10 mm		15 mm		18 mm	
		x_{mean}	x_{05}	x_{mean}	x_{05}	x_{mean}	x_{05}
tension strength							
major axis	$f_{x,t}$	19,8	14,8	15,5	13,0	18,2	16,4
minor axis	$f_{y,t}$	12,2	10,6	9,8	8,0	11,5	11,0
compression strength							
major axis	$f_{x,c}$	29,0	27,1	24,9	22,5	25,6	24,1
minor axis	$f_{y,c}$	19,2	17,7	16,0	14,0	18,4	17,0
shear strength	f_v	12,6	10,6	11,8	10,8	11,4	9,8



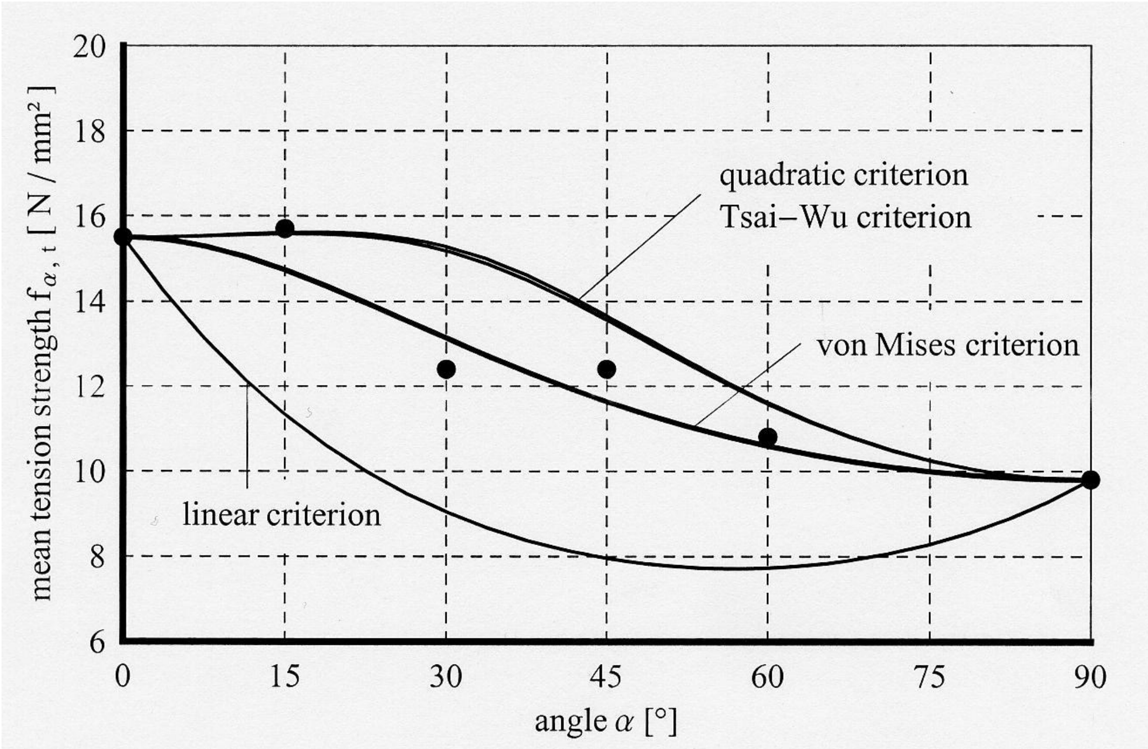
a)



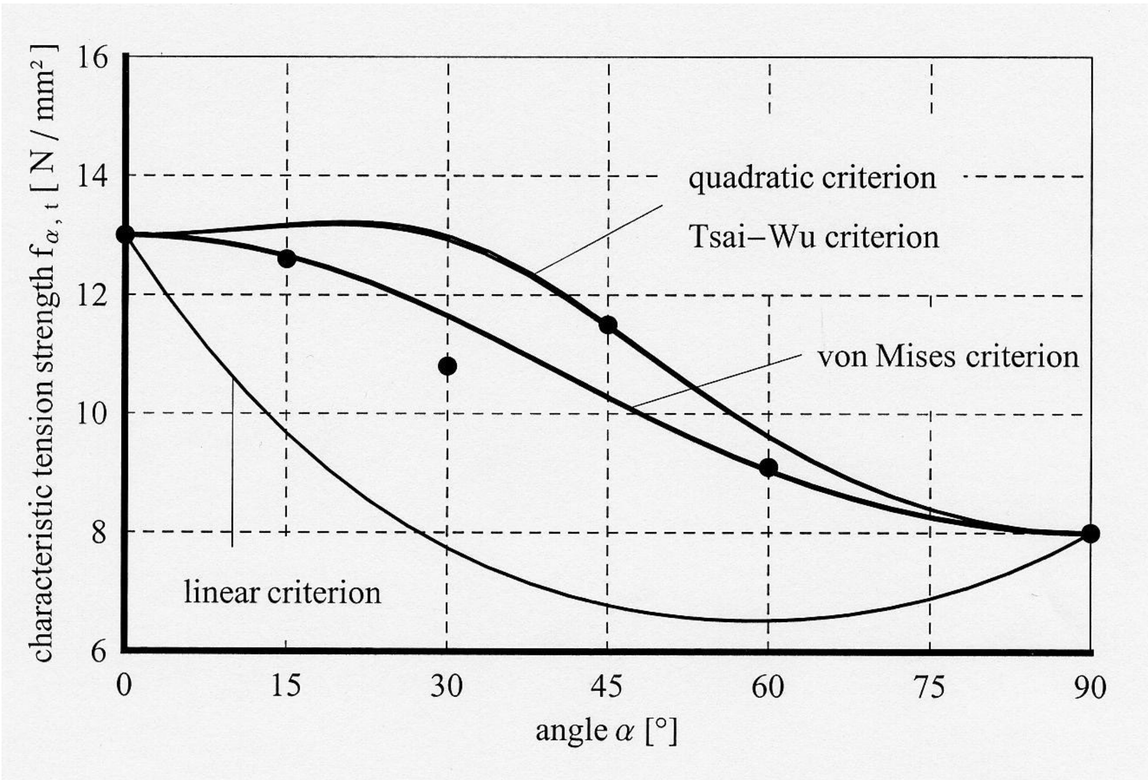
b)

Fig. 1a,b: *On- and off-axis tension strength of OSB/4 [N.N. 2000b] for a panel thickness of* 10 mm *and approximations acc. to four different strength criteria*

a) mean strength level *b) 5% fractile level*



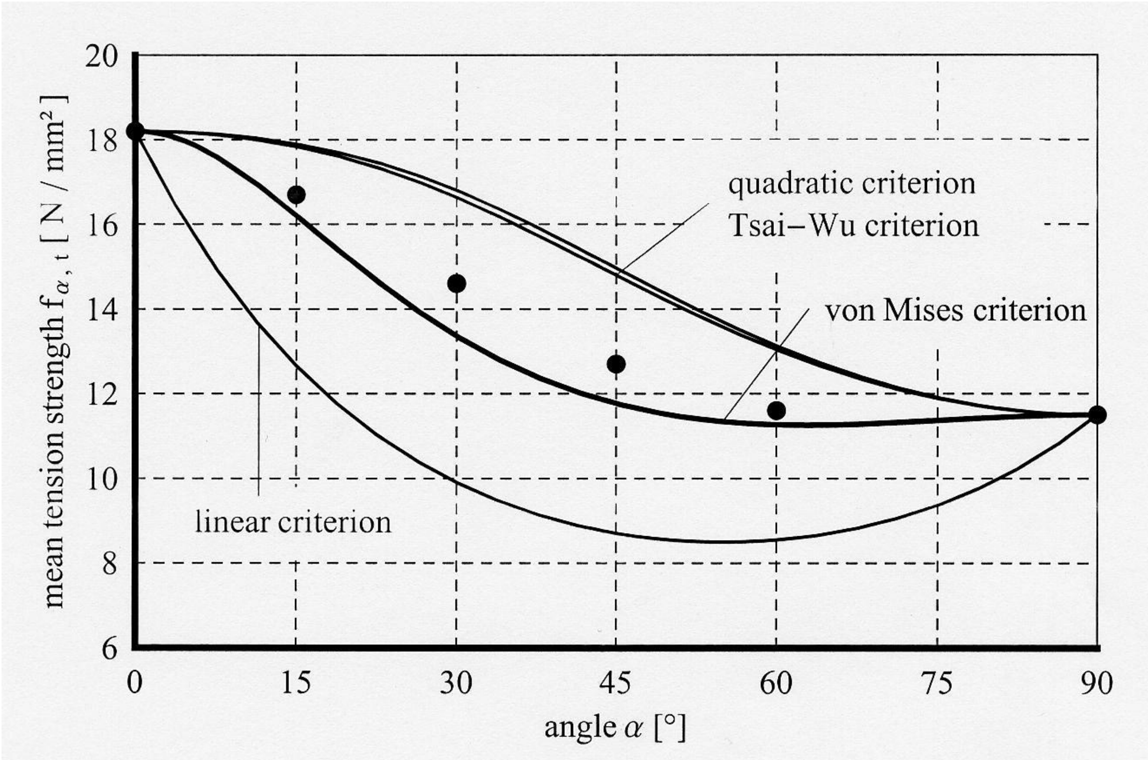
a)



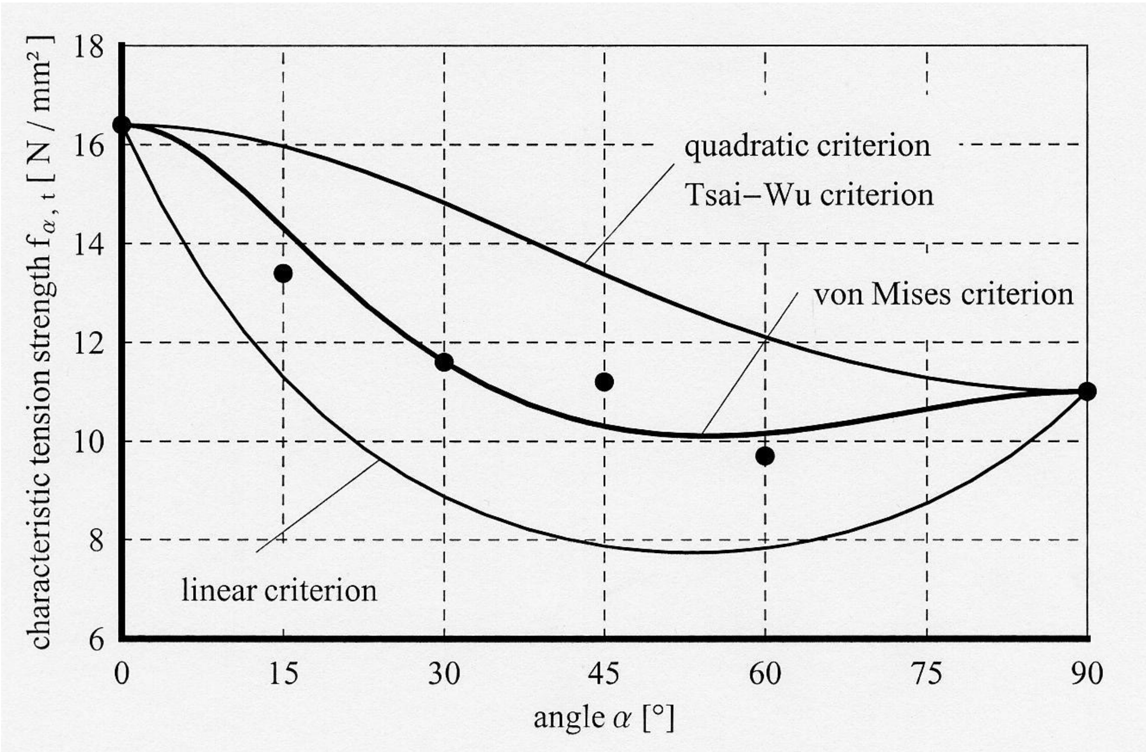
b)

Fig. 2a,b: *On- and off-axis tension strength of OSB/4 [N.N. 2000b] for a panel thickness of* 15 mm *and approximations acc. to four different strength criteria*

a) mean strength level *b) 5% fractile level*



a)



b)

Fig. 3a,b: *On- and off-axis tension strength of OSB/4 [N.N. 2000b] for a panel thickness of* 18 mm *and approximations acc. to four different strength criteria*

a) mean strength level *b) 5% fractile level*

Figures 1a,b to 3a,b show the computed off-axis strengths $f_{\alpha,t}$ depending on angle α acc. to the discussed different approaches on the mean and characteristic strength level, respectively. Further given are the experimental off-axis strength values. Regarding all graphs, the following statements concerning the aptness of the different failure criteria for approximation of the experimental data sets for off-axis tension strengths can be made:

- the strength equation acc. to the linear criterion, as proposed in the draft for the new German timber design code, obviously is far too conservative and delivers a rather poor approximation of the experimental data. It was shown that it is an intrinsic property of this equation to have an extreme value within the off-axis angle range; this feature was not consistently observed in the regarded experimental data sets.
- the quadratic failure criterion and the Tsai-Wu criterion enable throughout a considerably better approximation of the data. However, concerning design, the approximation of the 5% fractile results is in general on the unsafe side.
- the best data approximation, both on the mean and the characteristic strength level, is obtained with the von Mises invariant criterion. The hereon based off-axis strength equation throughout gives an almost ideal fit of the test results. It is important to state that the von Mises based strength equation is tendentially on the safe side what is especially important for the fit of the 5% fractile values.

It is noteworthy to state that all failure criteria based either on stress tensor invariants (Hankinson and von Mises invariant criterion) or on invariance vs. co-ordinate transformation (Tsai-Wu criterion), although purely hypothetical, reveal considerable better approximations of the experimental data sets as strength approaches without invariant character. In the performed study, this superiority was most evident for the criterion based on the von Mises invariant.

The authors propose for the revisement of the draft of the German timber design code the replacement of the off-axis strength equation based on linear interaction by the strength equation based on the von Mises invariant. This leads to a considerable increase of the competitiveness of wood-based panels in case of off-axis loading.

ACKNOWLEDGEMENTS

The authors owe gratitude to their colleague Helmut Rohlfing, in charge of the OSB approval, for providing the experimental data. Cordial thanks are indebted to Patrick Castera from Laboratoire du Bois de Bordeaux (LRBB) for his repeated favour in performing the translation of the French abstract.

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