BASIC CONSIDERATIONS TO ROLLING SHEAR MODULUS IN WOODEN BOARDS

ELEMENTARE BETRACHTUNGEN ZUM THEMA ROLLSCHUB-MODUL IN HOLZBRETTERN

ÉLÉMENTS DE BASE SUR LE MODULE DE CISAILLEMENT DE PLANCHES EN BOIS

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SUMMARY

The paper reports on numerical calculations concerning the so-called rolling shear modulus in wooden boards, i.e. the global shear stiffness of macroscopic cross-sections of boards subjected to shear forces perpendicular to fiber direction. The presented preliminary investigation demonstrates that rolling shear modulus is not an intrinsic material property but an apparent smeared shear stiffness of structural elements depending on several elastic coefficients, geometry and size parameters besides the mesoscopic on-axis in-plane shear modulus $G_{RT}$. Exemplarily, the pronounced dependency on sawing pattern, defining the location of the individual cylindrical material coordinate system of the board, is evaluated quantitatively by means of Finite Element Method.

ZUSAMMENFASSUNG

RÉSUMÉ

L'article traite de simulations numériques du module de cisaillement de type 'rolling shear' de planches de bois, i.e. la rigidité globale en cisaillement de sections droites de planches soumises à un cisaillement perpendiculaire au fil du bois. L'étude préliminaire présentée montre que ce module de cisaillement n'est pas une propriété intrinsèque du matériau. Le module apparent global en cisaillement d'éléments structuraux dépend de nombreux paramètres d'élasticité, de taille et de forme en plus du module mésoscopique de cisaillement $G_{RT}$. À titre d'exemple, un calcul par le Méthode des Éléments Finis a permis de quantifier la forte influence du type de débit de sciage définissant le repère en coordonnées cylindriques de chaque section.

KEYWORDS: rolling shear modulus, apparent or intrinsic material property, multilayer wooden elements, cylindrical anisotropy, influence of sawing pattern

1. INTRODUCTION

The issue of rolling shear stiffness and strength has attracted some consideration in the recent past alongside with the increasing use of multilayer solid wood elements. Multilayer solid wood elements are quasi cross-plys where the individual plys consist of boards arranged parallel to each other with rather small gaps. Each ply or board layer is rotated in-plane by 90 degrees vs. the layers above and below (Fig. 1). Typical built-ups consist of 3 and 5 layers [i. a. N.N., 1998] but there are also built-ups with 27 layers [i. a. N.N., 2000]. The elements, produced up to very large sizes of about $3 \text{ m} \times 16 \text{ m}$ are used for walls, roofs and bridge decks [i. a. SCHICKHOFER, 1998].

In bending out of plane with top and bottom layers oriented in span ($0^\circ$) direction, the boards of the in-between arranged 90° layers are subjected to so-called rolling shear. The magnitude of effective bending stiffness of the element and hence stress distributions in the individual layers depend on the magnitude of the rolling shear modulus of the 90° layers. A value of 50 N/mm² has been proposed by [BLAß AND GÖRLACHER, 2000] for this “elasticity” quantity of structural sized boards and this value has also been implemented in a general building approval for a respective multilayer solid wood element [N.N. , 2000].

The given magnitude of the rolling shear modulus seems very conservative as it is within the range of the on-axis radial-tangential shear modulus $G_{RT}$ on the mesoscopic scale (i. e. on the scale with negligible annual ring curvature).
In a profound investigation on mesoscopic material properties of spruce by [NEUHAUS, 1981] a $G_{RT}$ value of 42 N/mm² related to 12% moisture content and a density $\rho_{12}$ of about 460 kg/m³ was obtained via torsion tests. This value has been substantiated in a recent experimental diploma thesis forwarding a range of 40 to 50 N/mm² for a larger density range. Through inverse identification from macroscopic tension specimen a best fit value of about 50 N/mm² for a density of 460 kg/m³ was obtained by [AICHER ET AL., 2001]. Experiments based on the Josipescu method delivered a somewhat higher $G_{RT}$ magnitude of 58 N/mm² for moisture contents of 10-11% [DUMAIL ET AL., 2000].

The aim of the on-going investigation is to demonstrate that rolling shear modulus of wood in structural sizes, i.e. on the macro-scale, is not an intrinsic material property but an apparent substitute quantity of a structure, namely the board cross section, subjected to shear. Thus the apparent rolling shear modulus depends on the orthotropic on-axis stiffnesses but also to a pronounced extent on the size and cutting pattern of the board; the latter defines the origin of the cylindrical resp. polar material coordinate system.

Fig. 1 Schematic built-up of wooden multilayer elements; here exemplarily the cross-section of a 5-layer element is shown

2. DETERMINATION OF APPARENT ROLLING SHEAR MODULUS

The apparent rolling shear modulus in this first study was numerically evaluated by means of the model configuration shown in Fig. 2. The authors are aware that the model may not be the ultimate best due to the imposed boundary conditions which at the edges produce a slightly different response as compared to a real bending situation.
Nevertheless, the approach enables a good validation of the basic influences under consideration. So, apparent rolling shear modulus $G_r$ here was determined as

$$G_r = \frac{\tau}{\gamma} = \frac{\sigma_x \cdot t_0 \cdot t}{b \cdot \Delta u_x}$$

where displacement $\Delta u_x$ was evaluated as the average displacement along the wide board edge $b$, $\sigma_x$ is the line load at the edge of the upper lamella, $t_0$ is the depth of the upper lamella and $t$ is the depth of the investigated shear loaded board cross-section. The FE-analysis was performed with plane strain conditions using rectangular elements. The individual material coordinate systems of the elements were aligned acc. to the respective global system as illustrated in Fig. 3. The origin of the global polar coordinate system is defined by two dimensions

- $d =$ distance of pith vs. ‘right‘ board edge
- $e =$ eccentricity of pith vs. mid-width of board.
3. STUDIED CONFIGURATIONS

In this study geometry, size and ratios of on-axis stiffness coefficients were kept constant, so only the influence of the sawing pattern for a specific board cross-section was regarded. For orthotropic on-axis stiffnesses in the radial and tangential directions the following ratios were used: \( E_R : E_T : G_{RT} = 1 : 0.67 : 0.042 \) and \( \nu_{RT} = 0.2 \). The results given following are based on the absolute value \( G_{RT} = 50 \text{ N/mm}^2 \).

The cross-sectional dimensions of the investigated board subjected to shear were \( b = 150 \text{ mm} \) and \( t = 30 \text{ mm} \). The thickness and the orthotropic elasticity coefficients of the \( 0^\circ \) outer layers were assumed as \( t_0 = 20 \text{ mm} \), \( E_x = 13000 \text{ N/mm}^2 \), \( E_y = 500 \text{ N/mm}^2 \), \( G_{xy} = 700 \text{ N/mm}^2 \) and \( \nu_{xy} = 0.015 \).

Figure 4 shows the six exemplarily analyzed configurations of sawing patterns. Only the middle layer of the calculated 3-layer built-up is shown. The respective values of pith location parameters \( d \) and \( e \) are summarized in Tab. 1. Three of the six configurations (1 to 3) show idealized sawing patterns with year rings of negligible year ring curvature parallel, perpendicular and at an angle of 45° vs. the wider board edge. The three further configurations 4-6 show realistic built-ups with curved year rings. Configuration 4 represents a quite common sawing pattern without eccentricity and a pith distance equal to board depth. Configuration 5 represents an asymmetric built-up with small pith distance, however no pith within the cross-section; in configuration 6 a board including the pith is regarded.

![Fig. 4 Schematic overview of the analyzed sawing pattern configurations](image-url)
4. RESULTS

Table 1 and Fig. 5 present the obtained apparent rolling shear moduli. As can be seen, the orthotropic configurations 1 and 2 forward, as anticipated, the lowest $G_r$ values rather similar to the input value $G_{RT} = 50 \text{ N/mm}^2$. Ideally $G_r$ and $G_{RT}$ should be identical; the difference is related to the boundary conditions being under further consideration.

The highest apparent rolling shear modulus $G_r = 193 \text{ N/mm}^2$ is obtained for configuration 3 with a $45^\circ$ degree angle between annual growth rings and the board edges. Again this result makes sense in a qualitative way as the shear force transfer now acts in a truss-like system via the relatively stiff diagonals in the radial and tangential on-axis directions. This is illustrated qualitatively in Fig. 6 showing a principle stress vector plot with schematic vector orientation details along board width.

<table>
<thead>
<tr>
<th>Configuration No.</th>
<th>short description</th>
<th>pith distance $d$</th>
<th>eccentricity $e$</th>
<th>apparent rolling shear modulus $G_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>--</td>
<td>--</td>
<td>$[\text{mm}]$</td>
<td>$[\text{mm}]$</td>
<td>[N/\text{mm}^2]</td>
</tr>
<tr>
<td>1</td>
<td>'lying annual rings'</td>
<td>$d \to \infty$</td>
<td>$e = 0$</td>
<td>45.3</td>
</tr>
<tr>
<td>2</td>
<td>'standing annual rings'</td>
<td>$d = -t/2$</td>
<td>$e \to -\infty$</td>
<td>44.9</td>
</tr>
<tr>
<td>3</td>
<td>annual rings at $45^\circ$</td>
<td>$d =</td>
<td>e</td>
<td>\to \infty$</td>
</tr>
<tr>
<td>4</td>
<td>'normal' symmetric board</td>
<td>$d = 30$</td>
<td>$e = 0$</td>
<td>132.5</td>
</tr>
<tr>
<td>5</td>
<td>asymmetric board</td>
<td>$d = 10$</td>
<td>$e = -50$</td>
<td>117.4</td>
</tr>
<tr>
<td>6</td>
<td>board including pith</td>
<td>$d = -t/2$</td>
<td>$e = 0$</td>
<td>88.0</td>
</tr>
</tbody>
</table>

In case of configurations 4 to 6 with curved annual growth rings, the values of rolling shear modulus are between the extremes, as the cross-sections exhibit both, areas of zero (or small) angles between on- and off-axis coordinate systems and areas with $45^\circ$ annual ring orientations.
Due to highest portion of annual ring orientation of about 45°, configuration 4 shows the highest apparent rolling shear modulus of the investigated configurations with curved annual ring structure. In case of the board including pith (configuration 6) the existence of “lying” and “standing” annual rings leads to a smaller apparent shear modulus as compared to configurations without pith in the cross-section.

Through all presented configurations the smallest and largest calculated rolling shear values differed by a quite high factor of about 4. Within the configurations with pronouncedly curved annual rings a factor of more than 1.5 was found.
mainly compression stresses  
equal compression and tension stresses  
mainly tension stresses

Fig. 6 Principle stresses resulting from FE-calculation for sawing pattern configuration 3 (annual growth rings at 45°)

5. CONCLUSIONS

The following conclusions can be drawn at present from the on-going investigations: Rolling shear modulus of wood at the macro-level, $G_r$, is not an intrinsic material property but a global apparent quantity. The magnitude of $G_r$ depends on the three influences:

- meso-scale on-axis properties (including density and annual ring width influence),
- cutting pattern of the board, i.e. location of pith,
- size and geometry of board’s cross section.

Rolling shear modulus has a lower bound almost equal to the on-axis shear modulus in the radial-tangential growth plane. All realistic macro-scale configurations deliver considerably higher apparent stiffness values (increases up to a factor of about 4). Detailed quantitative results from a parameter study will be forwarded in a separate paper.

Rolling shear modulus for structural sized board cross-sections should be theoretically considerably larger than 50 N/mm², as assumed widely today in literature.
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