

CONCENTRATED LOAD ASPECTS OF SANDWICH PANELS WITH THICK ESPECIALLY WOOD BASED FACES

EINFLUSS VON EINZELLASTEN BEI SANDWICHPLATTEN MIT DICKEN DECKSCHICHTEN INSBESONDERE AUS HOLZWERKSTOFFEN

ASPECTS DES CHARGES CONCENTREES AUX PANNEAUX SANDWICH AVEC DES RECOUVREMENTS EPAISSES ESPECIALEMENT A BASE DE BOIS

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SUMMARY

The paper deals with the influence of concentrated loads on stresses of 3layer sandwich panels with thick faces, the latter in this context representing wood-based materials. According to a respective DIN standard covering bending testing of sandwich elements, approximative membrane theory may be applied in case that the ratio of face to total bending stiffness does not exceed 1 %. The appropriateness of said criterion is questioned especially for stress evaluations within the influence area of concentrated loads in presence of thick faces.

As an example a two-span element loaded with single loads in each mid-span is investigated. It is shown, that in the vicinity of single loads, membrane theory delivers far too low outer fiber bending and face shear stresses compared to »exact« sandwich theory whereas approximative core shear stresses are on the safe side. It has to be mentioned that in practice, considering finite bearing areas for applied single loads or supports, the extreme discrepancy of the two computational approaches may be considerably less accentuated. Nevertheless, said standard 1 %-criterion for membrane sandwich theory application in case of thick faces and concentrated loads should be reflected upon.

ZUSAMMENFASSUNG

Der vorliegende Aufsatz befaßt sich mit dem Einfluß von Einzellasten auf die Spannungen 3schichtiger Sandwichelemente mit dicken Deckschichten, hier speziell aus Holzwerkstoffen. Zufolge einer DIN-Norm betreffend Biegeversuche an Sandwichelementen kann eine Näherungsberechnung basierend auf der sog. Membran-Sandwichtheorie vorgenommen werden, solange die Eigenbiegesteifigkeiten der Deckschichten kleiner 1 % der Gesamtbiegesteifigkeit sind. Die

Zutreffendheit dieses Kriteriums bei vergleichsweise dicken Deckschichten ist jedoch insbesondere für Spannungsberechnungen im Einflußbereich von Einzel-lasten in Frage zu stellen.

Beispielhaft hierzu wird eine Zweifeldplatte mit Einzellasten in den Feldmitten untersucht. Es läßt sich zeigen, daß die Membrantheorie im Bereich von Einzellasten wesentlich zu niedrige Biegerand- und Schubspannungen für die Beplankungen ergibt, während die Kernschubspannungen zufolge Näh-erungsberechnung auf der sicheren Seite liegen. Endliche Lastaufstandslängen, die in der Praxis durchweg gegeben sind, können abmessungsabhängig die extremen Unterschiede zufolge beider Berechnungsansätze deutlich abschwä-chen. Grundsätzlich sollte jedoch das 1%-Normkriterium betreffend Anwend-barkeit der Membran-Sandwichtheorie im Hinblick auf die Einzellastproblema-tik speziell bei dicken Deckschichten überdacht werden.

RESUME

Le travail présent décrit l'influence de charges concentrées sur les contraintes de panneaux sandwich à trois lames avec des couches de recouvrement épaisses, dans le cas présent, à base de bois. Selon d'un standard DIN concernant des essais de flexion sur d'éléments sandwich un calcul approximatif basant sur la théorie membranes-sandwich peut être réalisé à condition que les rigidités de flexion propres des couches de recouvrement n'excèdent pas 1 % de la rigidité de flexion totale. L'aptitude de ce critérium est mise en question spécialement pour des calculs de contraintes sous l'influence de charges concentrées en présence des recouvrements épaisses.

Comme exemple un élément à deux travées avec des charges concentrées aux milieux des travées a été examiné. Il a été montré que la théorie de membrane dans les domaines de charges concentrées produit des contraintes de flexion et de cisaillement beaucoup trop basses dans les panneaux extérieurs en comparaison à la théorie »exacte«, tandis que les contraintes de cisaillement approximatives du cœur sont à côté sûr. Il doit être mentionné qu'en pratique, en tenant compte de domaines portantes finies pour les charges concentrées, la divergence énorme entre les deux calculs peut être considérablement atténuée. Néanmoins le critérium standard de 1 % concernant l'applicabilité de la théorie membranes-sandwich devait être reconsidéré en vue des couches plus épaisses et des charges concentrées.

Key-words: Sandwich elements, thick wood-based material faces, continuous spans, concentrated load influence, (membrane) sandwich theory

1 INTRODUCTION

Sandwich panels with faces from wood-based materials and a core of expanded foam offer an interesting potential as building construction material, i. a. apt for load bearing walls in prefabricated houses and roofs in general. The serviceability of such panels has been proven in the field for several years in the United States and some European countries; in Germany, due to different building tradition, an increasing trend for respective employment can be stated now. As wood-based sandwich panels are not covered by the German timber design code DIN 1052 such constructions are subject to the general building approval procedure.

It is reported hereinafter on a specific issue, relevant in current approval tests resp. evaluations conducted with continuously manufactured sandwich elements made from 16 mm thick chipboard faces and a 110 mm in-situ foamed polyurethane core. The paper deals with computational aspects of concentrated loads resp. support forces applied to sandwich panels with thick faces.

2 SANDWICH TEST EVALUATION ACC. TO DIN 53293

German materials testing standard 53293 describes details of a four-point bending test of flat sandwiches intended for determination of strength and deformation characteristics of 3layer compounds built-up from deliberate core and face materials. Fig. 1 shows the test scheme and Fig. 2 denotes the general cross-sectional and material terms. Equal loads are applied at the quarter points and deflections have to be measured at the loading points and at mid-span.

Further test details specified are secondary in this context. For evaluation of the test results with respect to stresses and stiffnesses the standard gives a set of formulas, stating, that these are valid only for compounds with neglectible bending stiffnesses of faces and core, each versus individual centroidal planes. Following, for reasons of simplicity only compounds with equal face thicknesses and moduli of elasticity will be considered.

Insignificance of face and core stiffnesses $2 E_f I_f$, $E_c I_c$ according to [1] and adopted by DIN 53293 is assumed to exist, when the ratios of said stiffnesses versus so-called membrane sandwich (bending) stiffness $E_f I_s$ are each lesser equal 1 %. Employing the respective expressions for the second moments of area (panel width Δ b)

$$I_f = b t^3/12, \quad I_c = b c^3/12, \quad I_s = b t d^2/2, \quad (1a-c)$$

the 1%-criterion delivers two ratios given in the standard, being:

$$\frac{2 E_f I_f}{E_f I_s} \leq 0,01 \rightarrow d / t \geq 5,8 \quad (2a)$$

and

$$\frac{E_c I_c}{E_f I_s} \leq 0,01 \rightarrow \frac{E_f}{E_c} \frac{t}{c} \left(\frac{d}{c} \right)^2 \geq 16,7. \quad (2b)$$

With both ratios (2) satisfied, a test evaluation can then be performed, based on so-called membrane sandwich theory. The simplicity of membrane theory consists in the fact that moment and shear action resp. resulting deflections w_M and w_Q can be regarded separately. This means, no sandwich theory in that,

only bending theory including shear – both confined to different discrete layers – could be applied. An individual deflection w along panel length ℓ may thus be easily computed from

$$w = w_M + w_Q = \frac{1}{E_f I_s} \int_0^{\ell} M\bar{M} \, dx + \frac{1}{G_c A_s} \int_0^{\ell} Q\bar{Q} \, dx \quad (3)$$

where

$$A_s = b d^2/c. \quad (4)$$

Expression (4), denoting the shear area of the core, is not completely evident at first sight, therefore see for instance [1].

Apart from deflection computation with known elasticity constants resp. stiffnesses $E_f I_s$, $G_c A_s$, eq. (3) enables an easy determination of both may be unknown stiffnesses via two experimentally measured deflections at different locations. Evaluating eq. (3) for the system in Fig. 1 (with stiffnesses left undetermined and renamed: $(EI)_{\text{eff}} \sim E_f I_s$, $S_{\text{eff}} \sim G_c A_s$), say at locations $x = \ell/4$ and $x = \ell/2$, and introducing the experimentally measured values $w_1 = w(\ell/4)$, $w_2 = w(\ell/2)$ into the left side of eq. (3) delivers

$$w_1 = \frac{F \ell^3}{96 (EI)_{\text{eff}}} + \frac{F \ell}{8 S_{\text{eff}}}, \quad (5a)$$

$$w_2 = \frac{11 F \ell^3}{768 (EI)_{\text{eff}}} + \frac{F \ell}{8 S_{\text{eff}}} \quad (5b)$$

and hence

$$(EI)_{\text{eff}} = \frac{F \ell^3}{256 (w_2 - w_1)}, \quad (6a)$$

$$S_{\text{eff}} = \frac{1}{\frac{w_1}{F \ell} - \frac{\ell^2}{12 (EI)_{\text{eff}}}}. \quad (6b)$$

The bending stresses for a given, may be ultimate moment in membrane theory are as usual $\sigma = M z/I_s$. Now, strictly, according to neglected bending stiffnesses of the faces, σ should only be evaluated for the face centroids $z = \pm d/2$, giving the average or membrane face stresses $\sigma = \sigma_m$. In an approximation, however, also the outer fiber stresses $\sigma = \sigma_r$ at cross-sectional depths $z = \pm h/2$ can be deduced:

$$\sigma_r = \pm \frac{M}{I_s} \frac{h}{2}. \quad (7)$$

Finally, the core shear stress according to membrane theory, assuming a constant shear stress between the face centroids and thereof a linear decrease to zero values at the face surfaces, is

$$\tau_c = \frac{Q}{b d}. \quad (8)$$

Above eqs. (6–8) are those given in DIN standard 53293. Further, the formulas are mandatory for test evaluation of almost all sensible cross-sectional sandwich built-ups with wood-based panels as faces and a polyurethane or polystyrene

core, as these compounds in majority obey the decisive ratios in eqs. (2). A critical review of the appropriateness of eq. (2a) is the very issue of this paper.

The neglect of the bending stiffnesses, resulting in a striking reduction of the computational effort was, as specified, based on a certain limiting 1 % ratio of face to membrane or approximately total bending stiffness. This, however, is only very crude, as the ratio of the approximative membrane versus the »exact« solution, especially for the face stresses, apart from bending stiffness ratio is further strongly depending in the statical system and thereby then on the location along panel length where stresses are computed. Additionally the ratio of bending to core shear stiffness has a distinct influence on validity of approximative membrane solution. So, for instance in case of the standard four point bending scheme, the error in approximative bending stresses (eq. (7)) versus »exact« solution for cross-sectional built-ups similar to Fig. 3 is nearly zero at mid-span but can well be in the range of 50 % to 80 % at the loading points. Following, after some annotations to »exact« sandwich theory, the above sketched error potential in vicinity of concentrated loads is demonstrated in detail for an – admittedly not standard covered – specific case of a continuous panel.

3 BASIC EQUATIONS OF »EXACT« SANDWICH THEORY

The basic equations of »exact« sandwich theory are given in very concise manner in order to assist transparency of hereinafter given computational results. »Exact« sandwich theory, as meant here, includes the influence of

bending stiffnesses of the faces, but still neglects the bending resistance of the core, the latter being of real minor influence to face stresses in case of the core and face materials regarded. Further, transverse incompressibility of the core and rigidity of the faces with respect to shear are assumed. For reasons of simplicity only symmetric cross-sections and no external inplane loading will be considered.

Fig. 2 shows an undeformed and deformed infinitesimal sandwich element with the characteristic partial section moments resp. forces of the discrete sandwich layers acting at the left cross-section and the equilibrating total section quantities M , Q at right hand side. Evidently, the total bending moment is split up in three components, being a pair of face moments M_f due to face bending stiffnesses and the typically dominating sandwich or membrane moment M_s , resulting from a pair of normal forces N_f , located in the face centroids:

$$M = 2 M_f + N_f d = 2 M_f + M_s . \quad (9)$$

Similarly, the total shear force Q can be written as

$$Q = 2 Q_f + Q_s \quad (10)$$

and further, what may also be taken from Fig. 2, M and Q alike usual beam theory are coupled through the moment and force equilibrium equations

$$M' = Q , \quad Q' = - q . \quad (11a, b)$$

In order to resolve the internally statically indeterminate system, two kinematic quantities are needed, normally being the overall vertical deflection w and the horizontally measured relative displacement of the faces δ , measured between the face centroids (Fig. 2). Equivalent to displacement δ , the shear angle $\Gamma = \delta/d$ could be employed. The partial section moments and forces, in terms of displacements w , δ resp. derivatives thereof ($(...)' = d(...)/dx$) – after having introduced the constitutive laws for face and core material – can be written as ([2], [3]):

$$M_f = - E_f I_f w'' , \quad M_s = E_f I_s (\delta'/d - w'') , \quad (12a, b)$$

$$Q_f = M_f' , \quad Q_s = G_c A_s \delta/d . \quad (13a, b)$$

Inserting the expressions for total moments and forces (eqs. 9, 10) into the equilibrium eqs. (11) with regard to the expressions (12, 13) gives a set of coupled fourth order differential equations:

$$\delta'' - (G_c A_s/E_f I_s) \delta = d w''' , \quad (14a, b)$$

$$- (2 E_f I_f) w^{IV} + (G_c A_s/d) \delta' = - q .$$

Equations (14) can easily be decoupled by means of differentiation, delivering

$$- (2 E_f I_f) \delta''' + (E_f I \omega^2) \delta' = - d q , \quad (15a, b)$$

$$- (2 E_f I_f) w^{VI} + (E_f I \omega^2) w^{IV} = \omega^2 q - q''$$

where

$$I = 2 I_f + I_s, \quad \omega^2 = (G_c A_p)/(E_f I_p). \quad (16a, b)$$

Equations (14, 15) are the basic deformation differential equations of the »exact« one-dimensional sandwich bending problem without external normal forces, however, still neglecting the bending stiffness of the core and assuming transverse incompressibility ([3]).

The relevant stresses can be most easily expressed by means of the partial quantities. Then, the outer fiber bending stresses of the faces and the shear stresses in the face centroids are:

$$\sigma_r = \pm \left(\frac{M_s}{I_s} \frac{d}{2} + \frac{M_f}{I_f} \frac{t}{2} \right), \quad (17a)$$

$$\tau_f = \frac{Q_s}{2 b d} + \frac{3 Q_f}{2 b t} \quad (17b)$$

and the core shear stress evolves as:

$$\tau_c = \frac{Q_s}{b d}. \quad (17c)$$

With respect to the first term of eq. (17b) and eq. (17c) compare annotation to eq. (8).

4 CONTINUOUS SANDWICH PANEL

A continuous sandwich panel as given in Fig. 3a, with two equal fields and concentrated (single) loads in the middle of each span, shall be regarded. The system and the specific dimensions reflect a configuration actually tested at Otto-Graf-institute in a current approval testing of sandwich panels. All equations and likewise the charts for forces, moments and stresses given hereinafter were computed via force method and superposition from displacement equations δ , w given in [4] for a simply supported one span panel loaded with a variable single load.

4.1 Reaction forces

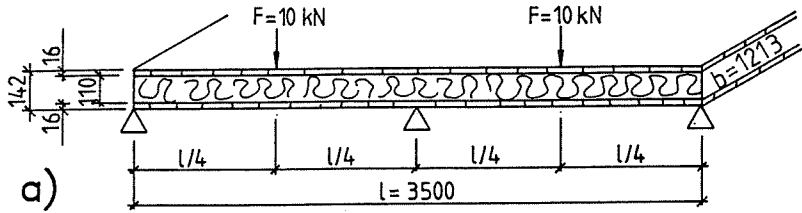
The expression for the vertical middle support force B_v , in general terms, results in

$$B_v = F \frac{\frac{11 \ell^3}{32} + \frac{3 \alpha^2 \ell}{\omega^2} - \frac{24 \alpha^2}{\lambda \omega^2} \frac{\sinh \lambda \ell/4 \sinh \lambda \ell/2}{\sinh \lambda \ell}}{\frac{\ell^3}{4} + \frac{3 \alpha^2 \ell}{\omega^2} - \frac{12 \alpha^2}{\lambda \omega^2} \frac{\sinh^2 \lambda \ell/2}{\sinh \lambda \ell}} \quad (18a)$$

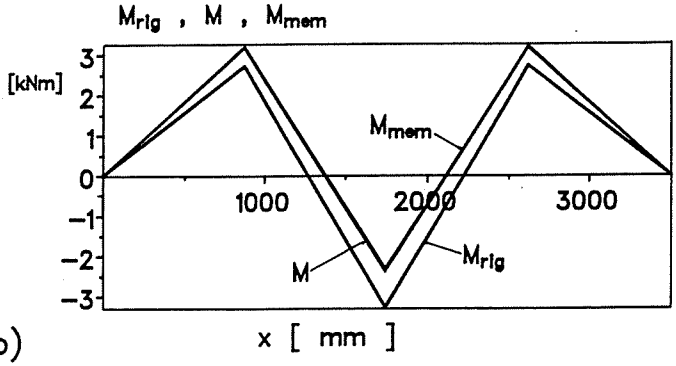
where

$$\alpha^2 = 1 - \beta^2 = I_y/I, \quad \lambda = \omega/\beta. \quad (19a, b)$$

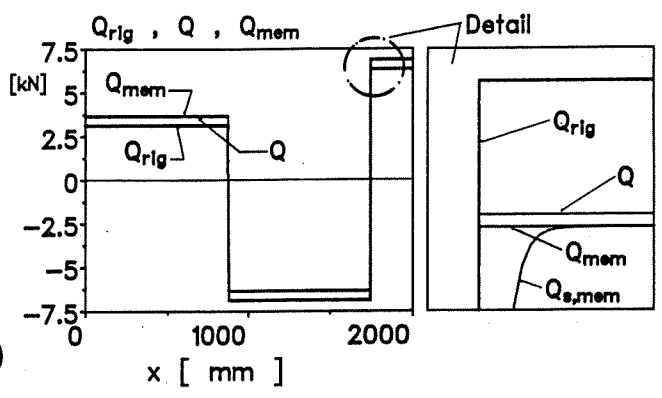
For the comparisons conducted, also the approximative membrane solution $B_{v, mem}$ as well as the solution for the shear rigid compound $B_{v, rig}$ are of interest and both can be easily derived from eq. (18a). In case of membrane



a)



b)



c)

Fig. 3a-c Investigated continuous sandwich panel with two equal spans and concentrated loads in each mid-span. Solutions for total section moments and shear forces acc. to »exact« and membrane sandwich theory resp. »usual« shear rigid beam theory
 a) dimensions, b), c) moments resp. shear forces

theory $\beta^2 \rightarrow 0$ and $\alpha^2 \rightarrow 1$, therefore $\lambda^2 \rightarrow \infty$ and hence the third terms of numerator and denominator in eq. (18a) vanish, delivering

$$B_{v, \text{mem}} = F \frac{1,375 \ell^2 + 12/\omega^2}{\ell^2 + 12/\omega^2} . \quad (18b)$$

An identic expression for eq. (18b) may be received without using sandwich theory but just employing eq. (3), i. e. beam theory including shear. Finally, for a shear rigid compound (G_c and $\omega^2 \rightarrow \infty$) with bending stiffness $E_f I_s$ (»usual beam theory«), eq. (18b) immediately delivers

$$B_{v, \text{rig}} = F \cdot 1,375 . \quad (18c)$$

Qualitative differences between the three solutions are given below along with the total section shear forces.

4.2 Total and partial section moments resp. shear forces

Fig. 3b shows the total section moments along panel length (panel dimensions see Fig. 3a) according to »exact« sandwich theory, membrane sandwich theory and shear rigid compound (»usual« beam theory) – M , M_{mem} and M_{rig} respectively – using reaction forces B_v given in eqs. (18a-c). Fig. 3c illustrates the corresponding shear forces Q , Q_{mem} , Q_{rig} ¹⁾. The results in Figs. 3, alike all further ones, are based on elasticity values $E_f = 2000$ Mpa, $G_c = 4$ MPa.

- 1) In case of membrane theory solution it should be emphasized that Q_{mem} given at left hand side of Fig. 3c results from the equilibrium condition of external vertical forces. When computing Q_{mem} from eq. (13b) as $Q_{\text{mem}} \triangleq Q_{s, \text{mem}}$ an equilibrium discrepancy compared to externally determined Q_{mem} occurs in the immediate vicinity of the concentrated loads, what is illustrated at right hand side of Fig. 3c.

As can be seen, there is a clear difference between section moments resp. shear forces according to »usual« beam theory and sandwich theory. However, the differences between membrane and »exact« sandwich theory, with respect to **total** section moments and shear forces, are minor. Generally, the taking into account of shear deformations causes a discharge of the middle support, compared to »usual« beam theory, resulting in increased field moments and hence in reduced negative section moments and shear forces at the middle support. For the specific configuration regarded, the differences of interest are: $\max M/\max M_{rig} = 1,16$, $\min M/\min M_{rig} = 0,73$, $\max Q/\max Q_{rig} = 1,16$ and $\min Q/\min Q_{rig} = 0,93$.

The minor differences in **total** section moments and shear forces between »exact« and **membrane** sandwich solution might mislead to the guess that these differences are similarly small in case of the stresses. In order to disprove this, first attention has to be paid on the split-up of total section moments and shear forces M , Q according to »exact« sandwich theory into the related partial terms M_s , $2 M_f$ resp. Q_s , $2 Q_f$. The partial section quantities are received from eqs. (12, 13). Figs. 4 and 5 illustrate the split-up for moment and shear force (in terms of absolute and relative numbers). In detail, Fig. 4a shows the total and partial quantities M resp. M_s , M_f along panel length and Fig. 4b gives the significant ratio $2 M_f/M$. Evidently, the typical sandwich moment M_s is steady differentiable at the locations of the concentrated loads including middle support force B_v . Contrary hereto, the skin bending moments M_f , being quite neglectible quantities at certain distances from the locations of the single loads rise to significant size in closer vicinity of the loading points and have a sharp bend there. In case of the investigated configuration, the ratio $2 M_f/M$ given in

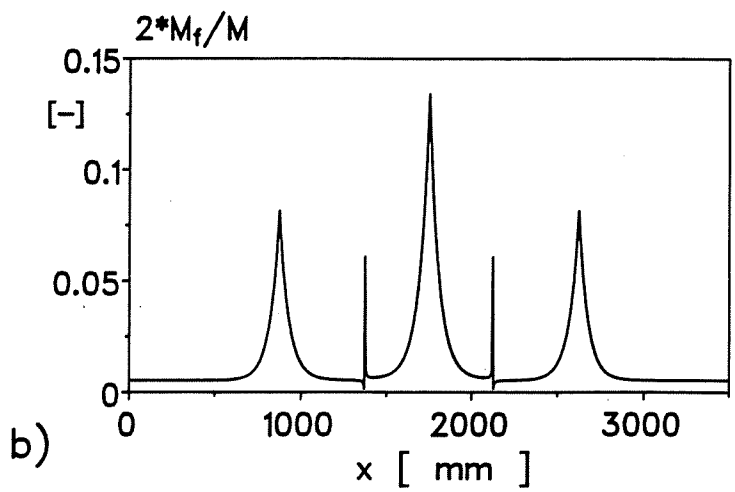
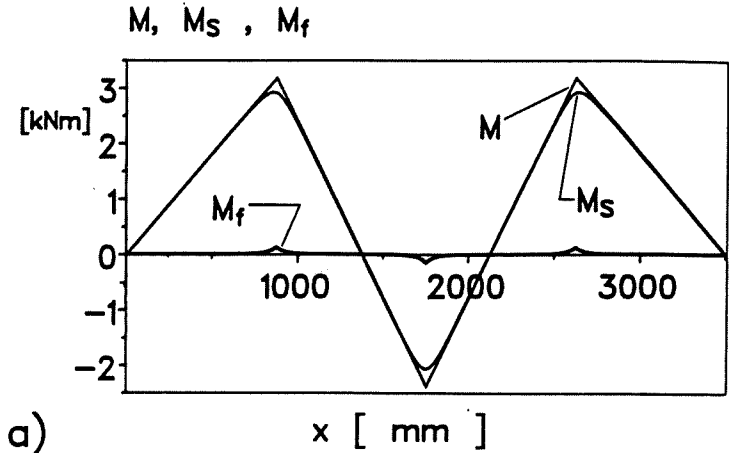


Fig. 4a, b Total and partial section moments of continuous panel (Fig. 3a) along spans according to »exact« sandwich theory
 a) M, M_s, M_f [kNm] b) ratio $2 M_f / M$

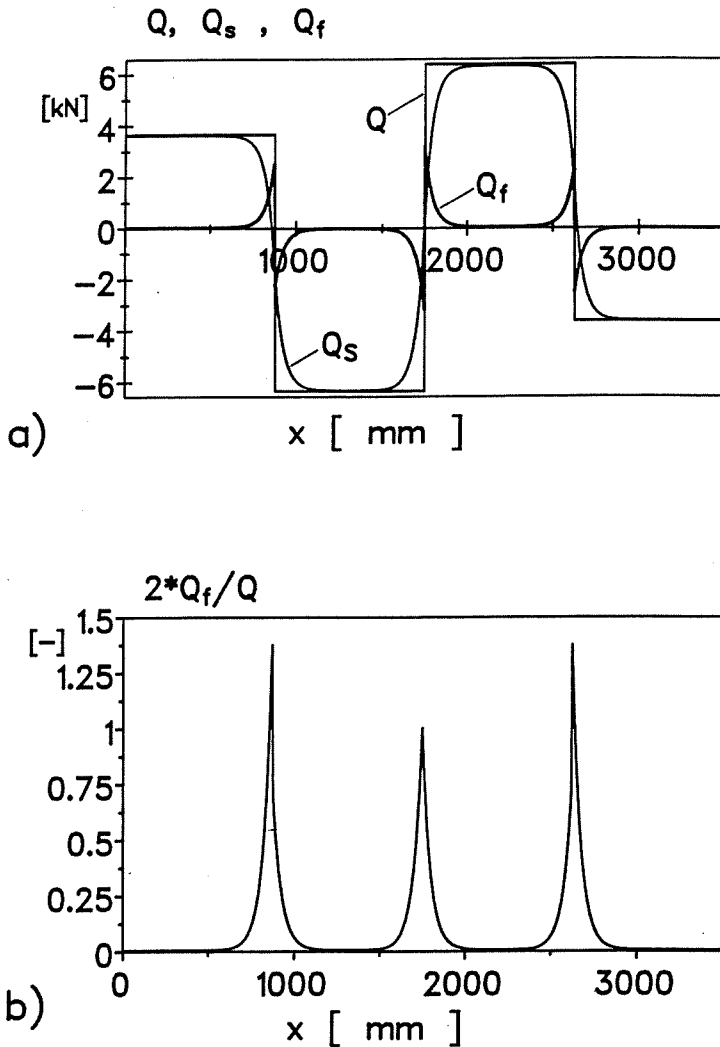


Fig. 5a, b Total and partial shear forces of continuous panel (Fig. 3a) along spans according to »exact« sandwich theory
 a) Q, Q_s, Q_f [kN] b) ratio $2 Q_f / Q$

Fig. 4b indicates that the skin moments carry 8,1 resp. 13,4 % of the total moment at the loading resp. support points.

The situation for the shear forces Q , Q_s , Q_f and the ratio $2 Q_f/Q$ (Fig. 5) is quite analogous and here not commented further.

4.3 Face bending and core shear stresses

The bending and shear stresses for »exact« sandwich theory are received by introducing the partial section quantities M_s , M_f , Q_s , Q_f (Figs. 4a, 5a) into the stress eqs. (17). The resulting outer fiber bending stresses σ_r at the bottom of the panel altogether with the corresponding stresses $\sigma_{r, \text{mem}}$ according to membrane theory (M_{mem} as in Fig. 3b, introduced in eq. (7)) are given in Fig. 6a. The chart shows that there is a significant stress discrepancy between »exact« and membrane sandwich theory solution in the areas of the concentrated loads (including support force B_v) which to that extent is hardly anticipated from the moment relations M , M_f resp. M_{mem} . Fig. 6b shows the ratio of approximative membrane versus the »exact« bending stress solution revealing that $\sigma_{r, \text{mem}}/\sigma_r$ at the loading resp. middle support locations is as low as 40 resp. 27 %, i. e. in design calculations these ratios correspondingly would represent underdesigns as high as 60 resp. 73 %. On the other hand in test data evaluation, say with bending stress failures in the load application areas, far too conservative bending stress capacities are deduced from the membrane theory approach.

Fig. 7a depicts the situation for the core shear stresses, constant over core depth, along panel length according to »exact« and membrane sandwich theory, τ_c and $\tau_{c, \text{mem}}$ respectively. Further, the »exact« shear stresses in the centroidal

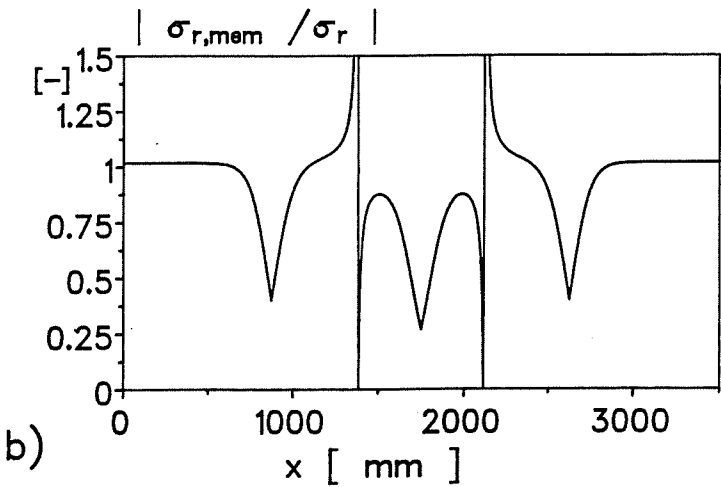
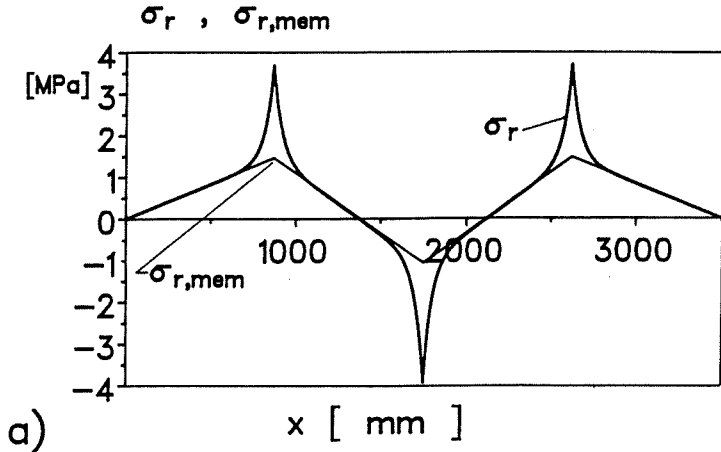


Fig. 6a, b Outer fiber bottom face bending stresses of continuous panel (Fig. 3a) along spans according to »exact« and membrane sandwich theory, σ_r resp. $\sigma_{r, mem}$
 a) $\sigma_r, \sigma_{r, mem}$ [MPa] b) ratio $|\sigma_{r, mem} / \sigma_r|$

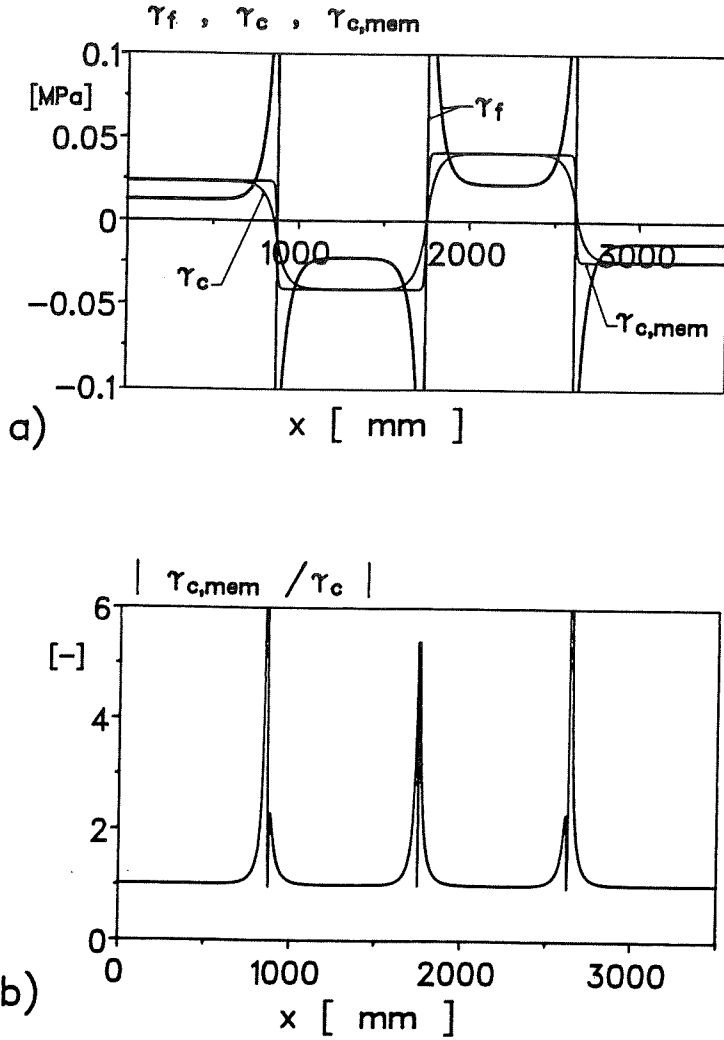


Fig. 7a, b Core shear stresses of continuous panel (Fig. 3a) along spans according to «exact» and membrane sandwich theory, τ_c resp. $\tau_{c, mem}$, and «exact» mid-depth face shear stresses τ_f
 a) $\tau, \tau_{c, mem}, \tau_f$ [MPa] b) ratio $|\tau_{c, mem}/\tau_c|$

planes of the faces are given. Fig. 7b illustrates the ratio $\tau_{c, mem}/\tau_c$. Alike and related to the bending stress variation along panel length, the core shear stresses τ_c and $\tau_{c, mem}$ are quite equal outside the loading resp. support locations. Then, in vicinity of the concentrated loads a divergence occurs, thus that membrane theory delivers higher stresses, contrary to above discussed outer fiber bending stress situation.

The face shear stresses τ_f at mid-depth of the faces are constant and of minor quantity at some distance of the concentrated loads. In closer vicinity thereto the τ_f values however rise to extreme 14 resp. 11fold magnitude at the load and mid-support points, mirroring the fact that at locations $Q_s = 0$ the total shear force Q is carried then by the faces. As zero values of Q_s and Q do not coincide at the loading points $x = l/4$ resp. $3l/4$, quantity $2Q_f$ there actually exceeds Q by factor 1,4 (compare Fig. 3a).

Now, in reality, the stress peak situation for bending and shear stresses of the faces in vicinity of concentrated forces is less problematic compared to the point load solutions given, as concentrated loads or support forces have discrete bearing areas, thus actually are line resp. surface loads. Then, the bending and shear stress peaks in the concentrated force areas reduce considerably compared to single point solutions, but nevertheless remain throughout higher compared to the membrane results. The extent of the stress discrepancies between »exact« and membrane solution, in case of thick non yielding wood-based and alike face materials are therefore related to the dimensions of the bearing areas which have to be taken into consideration in the computations.

5 CONCLUSIONS

Bending testing and test evaluation of sandwich panels with comparatively thick faces made from wood-based materials has to be performed according to building materials standard DIN 53293, describing a four-point bending test. Said standard, covering deliberate component materials, states that in case the ratio of face to approximately total bending stiffness does not exceed 1 %, test evaluation regarding stresses and strengths may be conducted by means of so-called membrane sandwich theory where bending stiffnesses of the faces are neglected. The influence of core stiffness can be regarded minor in this context, anyhow.

The paper questions the appropriateness of the above ratio as sole criterion, as the differences between solutions according to »exact« and membrane sandwich theory cannot be confined solely to said stiffness ratio, but also depend strongly on the statical system, the location of the stress evaluation along panel length and the ratio of face to core shear stiffness.

For demonstration of the effect of concentrated loads, also present in the standard test, a practice relevant sandwich panel continuous over two equal spans and loaded with single loads in each mid-span was investigated. The computations revealed that the outer fiber bending and mid-face shear stresses in vicinity of concentrated loads, idealized as single forces, are extremely underestimated by membrane compared to »exact« sandwich theory in case of thick faces. Contrary hereto, the core shear stresses due to membrane theory are throughout of equal magnitude resp. on the safe side.

In reality, concentrated loads possess certain bearing areas what, depending on the individual dimensions, can reduce the discrepancies between the approximative membrane and the »exact« sandwich theory solutions considerably. Nevertheless, the too vague 1 % stiffness ratio criterion of DIN 53293 for membrane theory application should be reflected upon, especially in case of thick faces and concentrated loads.

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