

**THE RESONANCE METHOD - APPLICATION OF A NEW
NONDESTRUCTIVE TECHNIQUE WHICH ENABLES
THICKNESS MEASUREMENTS AT REMOTE CONCRETE PARTS**

**DIE RESONANZMETHODE - ANWENDUNG EINES NEUEN
ZERSTÖRUNGSFREIEN PRÜFVERFAHRENS IM HINBLICK AUF
DICKENMESSUNGEN AN SCHWER ZUGÄNGLICHEN
BETONBAUTEILEN**

**LA METHODE DE RESONANCE - APPLICATION D'UNE
NOUVELLE METHODE DE MESURE NON-DESTRUCTIVE PAR
EGARD A MESURES D'EPAISSEUR DES ELEMENTS EN BETON
MAL ACCESSIBLES**

Christian U. Große, Hans W. Reinhardt

SUMMARY

The method of measuring the resonance frequencies of an object is comparatively new in the area of nondestructive testing techniques. In this method, waves produced by a short stress pulse and interfering in the specimen are analysed referring to their frequency spectrum. The resonance method is applied to structures that are not easy to access for testing, like concrete slabs and floors. We demonstrate that this method is capable of determining the thickness of these structures nondestructively and with sufficient precision.

This method may be transferred to other materials at different expenditures. Besides, easy determination of stress wave velocity is possible, if the dimensions of the object are known.

ZUSAMMENFASSUNG

Das Verfahren der Messung von Resonanzfrequenzen ist für den Bereich der zerstörungsfreien Werkstoffprüfung relativ neu. Bei dieser Methode werden die von einer Impulsanregung ausgehenden und im Körper interferierenden Wellen bezüglich ihres Frequenzspektrums analysiert. Ein Anwendungsgebiet für diese Resonanzmethode stellen Bauteile dar, die schwer (d.h. beispielsweise nur von einer Seite) zugänglich sind wie Betonplatten oder -böden. Es wird gezeigt, daß die Methode in der Lage ist, deren Dicke zerstörungsfrei und mit hinreichender Genauigkeit zu bestimmen.

Das Verfahren ist mit unterschiedlichem Aufwand auf andere Werkstoffe übertragbar. Außerdem ist bei bekannten Objektabmessungen eine einfache Bestimmung der Kompressionswellengeschwindigkeit möglich.

RÉSUMÉ

La méthode de mesure des fréquences de résonance est assez nouvelle dans le domaine d'essai non-destructif. Le principe de cette méthode est une analyse spectrale à ondes partantes d'une impulsion mécanique à la surface. Une application de cette méthode de résonance constituent les éléments en béton mal accessibles (qui, par exemple, sont accessibles seulement d'un côté). Ce sont des dalles et fondations en béton. Il est montré que cette méthode est capable de déterminer leur épaisseur sans détruire et avec une précision suffisante.

La méthode peut être transportée à renfort différent chez matériaux différents. En outre il est possible de déterminer la vitesse des P-ondes si l'épaisseur de l'objet est connue.

KEYWORDS: non-destructive testing; crack detection; concrete; impact-echo; resonance; wave propagation.

1. INTRODUCTION

First measurements of resonated waves produced by an impact on the surface of concrete structures were reported by Carino and Sansalone [1, 5, 6]. They developed the so-called "impact-echo method" in particular for the detection of ungrouted ducts and horizontal cracks in concrete. Although the phenomenon of waves reverberating between two discontinuity surfaces is well-known, until now a general transfer to material research is missing. An example is the geophysical prospection, where waveforms produced by vibroseismic sweeps and explosions are evaluated not only in time domain, but the reverberation modes are also analysed in frequency domain. Fig. 1 from Berkhemer [2] shows schematically the different reverberating modes that are caused by the bedding in the upper earth crust.

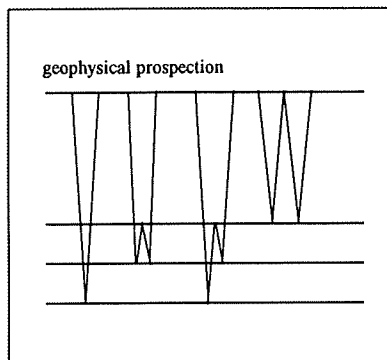


Figure 1: "Multiples" caused by reverberations between different layers in the upper crust (after Berkhemer [2])

Besides the detection of damages and inhomogeneities in an object, this technique is also capable of determining the dimensions of elements. This is a well-known problem, i. e. when making researches and expertises of damage at concrete buildings, especially when they are remote, like concrete pavements and catching basins. In these cases, today normally destructive tests are performed, using endoscopy in drill holes or examining drilling cores. In the conventional ultrasonic pulse-echo method, the stress pulse applied on the surface by an ultrasonic impactor is detected by a piezoelectric transducer. The return time of the wave reflected by an interface in the medium gives the information that is needed. In concrete, this method is failing due to the heterogeneity caused by the reinforcement and the large grain size of the aggregate. Additionally, the wave field radiated by the impactor is disturbing the transducer, so that all further impulses are vanishing in the coda of the primary wave. Attempts have failed to optimize this setup of measuring for the use in concrete [3].

The following studies were motivated by the wish of introducing a nondestructive testing method for these cases. Indeed, it has to be accentuated that this method, after fundamental optimization, also may be applied to other materials like natural stone, metals, wood. First experiences have been made using this new method for detecting spalls and flaws in statuettes of sandstone [9].

2. PHYSICAL BACKGROUND

The problem of wave propagation in concrete has already been discussed earlier [4]. Nevertheless, there are some new aspects with regard to the presented resonance method. Therefore, the physical background, already explained in detail by Carino and Sansalone [5] and Koo et al. [6], will be summarised for the present problem and illustrated by fig. 2.

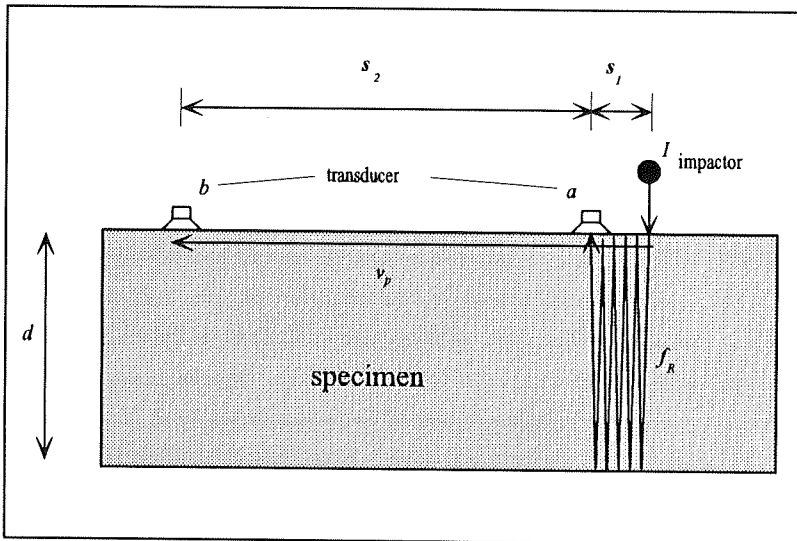


Figure 2: Principle of resonant measurements.

The wave field radiated by the impactor I is propagating into the object along spherical wavefronts. After a short travel time t_1 , the direct wave meets the transducer a . Depending on the distance between impactor and transducer, at a later time also the background echo reaches the transducer. However, this signal is vanishing in the

seismic recording of the primary wave due to the strong attenuation in concrete (15-20 dB/m). Besides, a waveform with period T (wavelength $\lambda_l = \text{double object thickness } 2d$) is developing, consisting of a constructive interference. This means, finally a stationary wave is dominating the recording, showing a rather low-frequency signal with frequency f_R . An analysis of the time signal by a fast fourier transform shows the frequency spectrum of the primary wave plus the resonance modes¹ according to the thickness of the object. This relation yields the following expression:

Equation 1:

$$d = \frac{v_P}{2 \cdot f_R}$$

In the case of additional discontinuities in the medium like spalls or horizontal flaws, the spectrum shows indeed additional maxima that are more or less clear. However, normally the main peak in the resultant spectrum is due to the background echo (discontinuity concrete-air).

This fact is illustrated in fig. 3. According to the method we will describe below, a wave has been applied on the surface of a concrete slab. This wave was detected with a piezo-transducer at a distance of 5 cm. Fig. 3 a) shows the complete signal in the time domain. The signal decays after approximately 20 ms. The transformation of this signal into the frequency domain² shows fig. 3 b). The main maximum in the amplitude spectrum near 6,7 kHz is corresponding (using eq. 1) to a thickness of the concrete slab of 30 cm and a wave velocity of 4020 m/s. The period of this wave is according to twice the object thickness.

¹ Besides waves with the fundamental mode f_R and the wavelength λ , there are additional waves interfering constructively: $(n \cdot f_R)$ and $n = 1, 2, 3, \dots$. The higher modes $n > 1$ are of no significance because of the attenuation.

² We used a Fast Hardley Transform (FHT) with 4092 points taped with a Hanning-window [8].

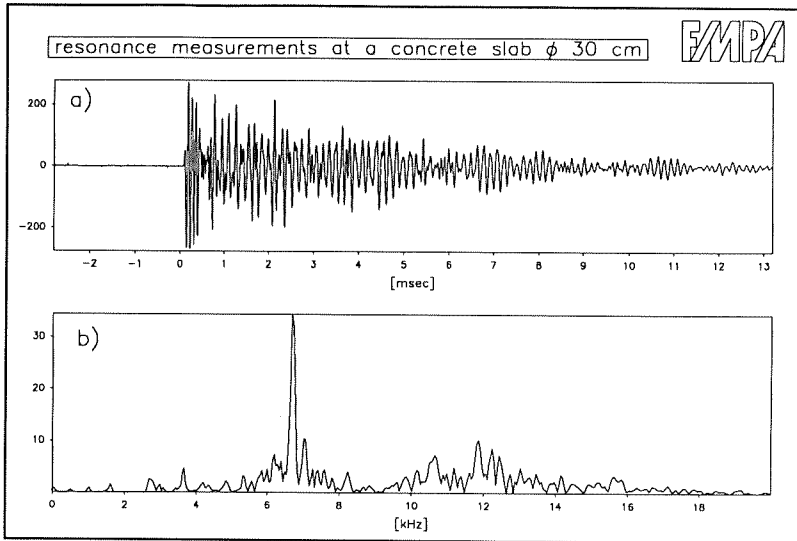


Figure 3: Calculating the thickness of a concrete slab with the resonance method.

3. MEASUREMENT PARAMETERS AND INSTRUMENTATION

As shown in chapter 2, the actual measured variable is the resonance frequency. But for using eq. 1, another variable has to be known, that is the P-wave velocity v_p or the depth of the discontinuity. If the latter isn't known, and this is the reason for our investigations, v_p has to be determined.³

3.1. Measurement of the P-wave velocity

Usually, the P-wave velocity in materials is determined by measuring the travel time of a triggered pulse. Yet the travel distance between impactor and transducer has to be known. In the case of remote objects, this method isn't applicable, since only the surface is accessible. At a little higher expenditure, it is possible to measure the P-

³ Knowing the dimensions of a specimen the method can be used for velocity measurements also.

wave velocity also on the surface of an object. The reason therefore is the propagation of the wave field also along the surface of the object. As shown in fig. 2, this measurement is enabled by an additional transducer at some distance from the other transducer. The signal produced by the impactor is triggering transducer *a* after a time t_1 and is reaching transducer *b*, travelling along the track s_2 . The travel time measured, the velocity easily can be computed:

Equation 2:

$$v_P = \frac{s_2}{t_2 - t_1}$$

During one *single* test, with the same impactor signal, it is possible to determine the P-wave velocity and simultaneously the object thickness. Thus it is possible to rewrite eq. 1 as:

Equation 3:

$$d = \frac{s_2}{2 \cdot f_R \cdot \Delta t} \quad \text{with} \quad \Delta t = (t_2 - t_1)$$

But as the travel path that is used for both measurements is not identical, the velocity measurement contains a systematic error. Chapter 4.4 gives information on the quantitative influence of the single sources of errors and their effects on the quality of the measurement.

We realized within the scope of our test measurements that also the distance between the two transducers has a considerable effect on the quality of the measurement. If the travel path is chosen too short, inhomogeneities under the surface of the concrete are affecting the measurement. This is shown in fig. 4, related to the experimental setup described in 4.1. Increasing error bars at decreasing distances of the transducers show the increasing scatter of the measured values. At about 25 cm the v_P -values have reached a certain constancy at 4020 m/s (± 100 m/s). For further measurements of

the compression wave velocity at the surface, thus the distance should not remain under this minimum.

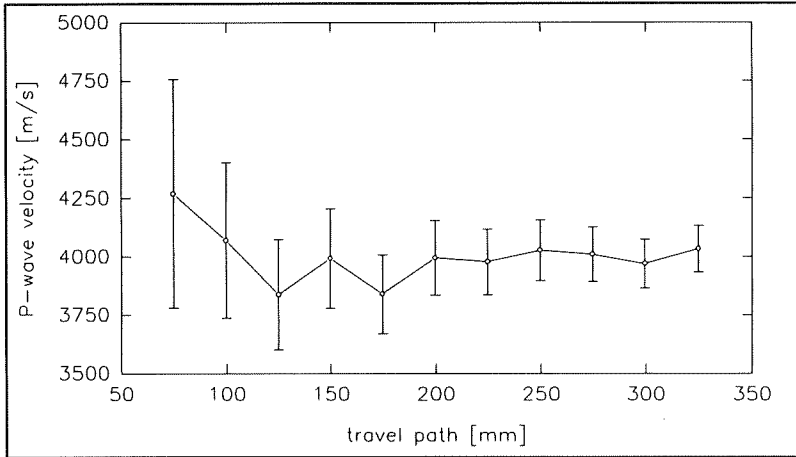


Figure 4: Variation of v_p depending on the distance s_2 between the transducers.

3.2. Impactor signal

As this non-destructive testing method uses sound waves, the excitation of these waves is of considerable importance for the success of the measurement. Especially required is a wide range of frequency components in the radiated wave field. This is the only way to make sure that the frequency corresponding to the thickness of the object is stimulated. This is easy to realize by dropping a small spherical steel ball from a certain height down to the surface next to the transducer. Besides it's obvious, that the width of the frequency spectrum is inverse to the contact time of the steel ball on the surface of the object - an idealized delta-peak is producing a white spectrum, the widest spectrum that is possible. Therefore, it is important to use steel balls as small as possible for the excitation. The larger the diameter of the balls, the lower-frequent are

the emitted signals, as shown in fig 5. Usually, we used steel balls with diameters of 4-8 mm. Different energies, respectively amplitudes of the impulse may easily be attained by varying the height of fall. Carino & Sansalone suggested to use a steel ball which is hit by a spring bolt as impact source. The advantage of this arrangement is a better control of the applied energy. But this parameter is not decisive for the aim of our measurements.

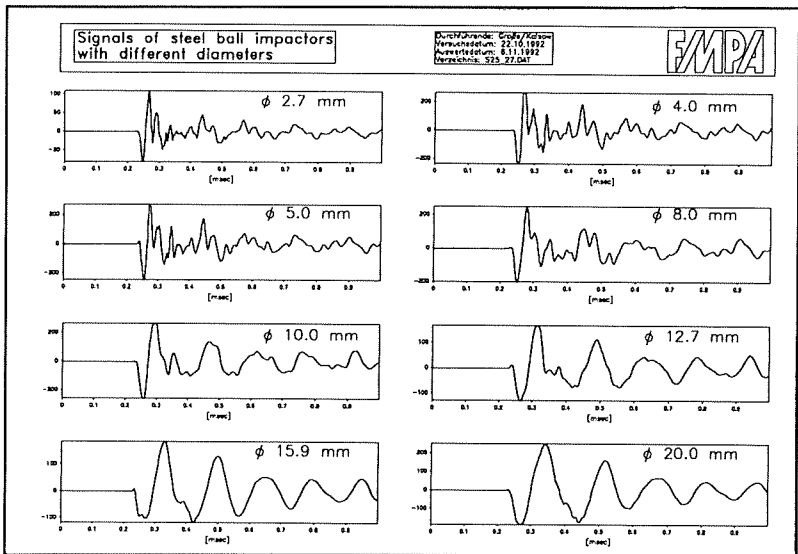


Figure 5: Shape of the impact signal depending on the steel ball diameter.

3.3. Signal recording

For the use of this measuring technique on structures, it is important to have a user's friendly and robust signal recording system. Also an instant check of the measuring results should be possible. For that reason, a direct A/D-conversion with the option of

a fourier transform is indispensable. As described earlier [4], we are using an 8-channel transient-recorder with a maximum sampling rate of 1 MHz per channel. Each channel has an own 12 bit A/D-converter. An additional use of preamplifiers is required only to improve the signal/noise-ratio. During our tests, the broad band piezo transducers were usually connected directly with the transient-recorder. A suitable software provided the computation of the fourier transform directly after the A/D-conversion of the signals. All results were displayed almost instantaneously on the screen. The data were stored first to the internal storage facility of the transient recorder (256 kByte RAM per channel). At a typical signal length of 2048 points, thus almost 100 single measurements can be recorded. Of course, it is possible to shift the measurement data to the 100 MByte hard disk drive.

4. RESULTS

For investigating the theoretical and physical interrelations in concrete, a series of tests were carried out. First, we performed laboratory tests on concrete plates for eliminating all parasitic inductions. Later, objects were tested under realistic conditions. At the first field experiment, we tested a retaining wall of reinforced concrete with known thickness. At the second one, a concrete floor in a hall was tested - the thickness was unknown. The results of both tests are presented below. Afterwards the quality of the measurement in general is estimated.

4.1. Measurements on different specimen

First the method has been tested on specimen in the laboratory. Two concrete plates of 30 cm thickness had been designed, one of them reinforced (plate B), the other plain (plate A). For avoiding resonance effects coming from the short sides of the plates, a rather large base (150x80 cm) had been chosen. Fig. 6 explains the experimental setup. The plates rested on three small wooden beams for reaching a clear

background echo. The point of the measurement was carefully chosen, thus an interface concrete-air was provided.

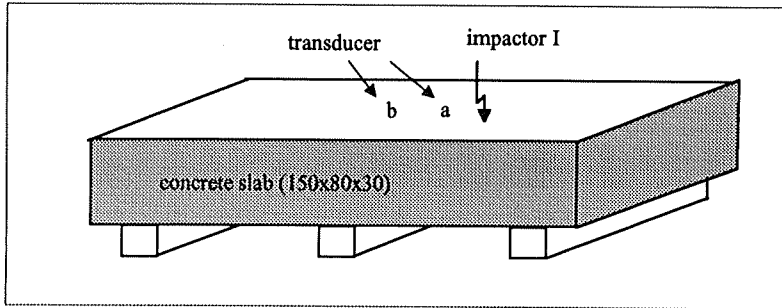


Figure 6: Concrete slab for laboratory tests.

First of all, the average P-wave velocity was determined by using the through-transmission pulse-velocity method (with an ultrasonic transmitter of 20 kHz resonance frequency). Altogether, 600 single tests at different points and with different transmitter-transducer configurations have been analysed. They are summarised in line 1 of tab. 1. The standard deviation is according to a relative error of only 3 %. Although we supposed a higher v_p -value in reinforced concrete, this could not be confirmed within measurement accuracy. The results presented in the second line we obtained by measuring v_p at the surface of the two plates. It was found that these velocities were distinctly slower than the ultrasonic pulse velocities.

The measurements of the frequency spectrum did not present similar difficulties. An example is shown in fig. 3. The frequency maximum of every fourier spectrum has been extracted and is shown in line 3 of tab. 1. The frequency values were remarkably reproducible and their scatter was far below the measuring accuracy of the speed

measurement. The thickness of the concrete plate is computed using eq. 3 and referring to the speed measured on the surface of the plates (line 4).

Table 1: Measurements of the thickness and the P-wave-velocities at two different slabs.

		A: not reinforced slab	standard deviation	B: reinforced slab	standard deviation
1	v_p (ultrasonic pulse) [m/s]	4179	± 113	4264	± 133
2	v_p (measured on the surface) [m/s]	3838	± 44	4063	± 51
3	resonance frequency [Hz]	6500	± 25	6875	± 25
4	thickness (acc. eq. 1) [cm]	29.6	± 0.5	29.5	± 0.5
5	v_p (with resonance method) [m/s]	3900		4125	

The ultrasonic pulse velocities are rather high compared to those obtained by measuring on the surface of the plates. The bleeding in the upper areas of concrete plates, this means the segregation of the aggregate grains during the compaction process, may be a reason for this discrepancy. For our interests, this effect seems to be equalized by the fact that in the resonance measurements occur slower velocities as well. Thus we are able to calculate the "correct" thickness (30 cm). Going the opposite way, this is easy to see: presuming the thickness as well-known, the P-wave velocity can be calculated with high precision, as shown in line 5. It is nice to see that these v_p -values correspond to those measured on the surface. An explanation therefore is hard to find. Already Carino & Sansalone [6, page 137; 1, page 117] have reported this effect. According to their researches, the difference between the two measurement methods comes to approximately 10 % of the ultrasonic pulse velocity. It was important for our considerations to see that the velocities measured on the surface produced quite useful results which lead to very exact values of the plate thickness. This fact was confirmed during the following field tests.

4.2. Tests at a retaining wall with the interface concrete-soil

The experimental setup described in chapter 4.1 is little exemplary as far as the interface concrete-air represents an idealized case with a high reflectivity. In practice, we have to deal with a different medium adjoining to the remote side of the concrete object. Because of that, we performed another experiment at a retaining wall of reinforced concrete. The thickness of the wall could be measured on the top side with values between 23,5 and 24,5 cm. In tab. 2 the results are shown. Once more, the P-wave velocity has been determined both using the transmission method and measuring it on the surface. The resonance frequencies have been measured on five different points. We computed the thickness of the wall using v_p measured on the surface (line 2) as well as the pulse velocity (line 3). The five power spectra are presented in fig. 7, giving an impression of the analysis of the resonance peaks. The similarity of the spectra is easy to recognize. The variations of the main frequency are probably representing variations of the thickness differing from point to point. The absolute values of the thickness are determined exactly according to eq. 1, although the v_p -values measured on the surface are distinctly slower than pulse velocities. Obviously the determination of the v_p -velocity on the surface contains an systematic error of the same order (see above).

Table 2: Measurements at a retaining wall.

		1. measure.	2. measure.	3. measure.	4. measure.	5. measure.	mean	v_p [m/s]
1.	resonance frequencies [Hz]	8813	8875	8688	8625	9000	8800	
2.	thickness [cm] (v_p surface)	23.3	23.1	23.6	23.8	22.8	23.3	4105
3.	thickness [cm] (v_p direct)	25.6	25.5	26.0	26.2	25.1	25.7	4520

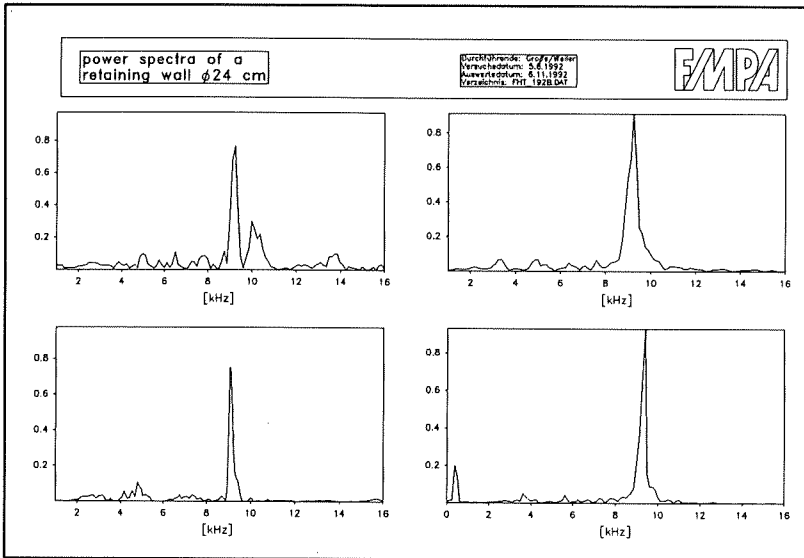


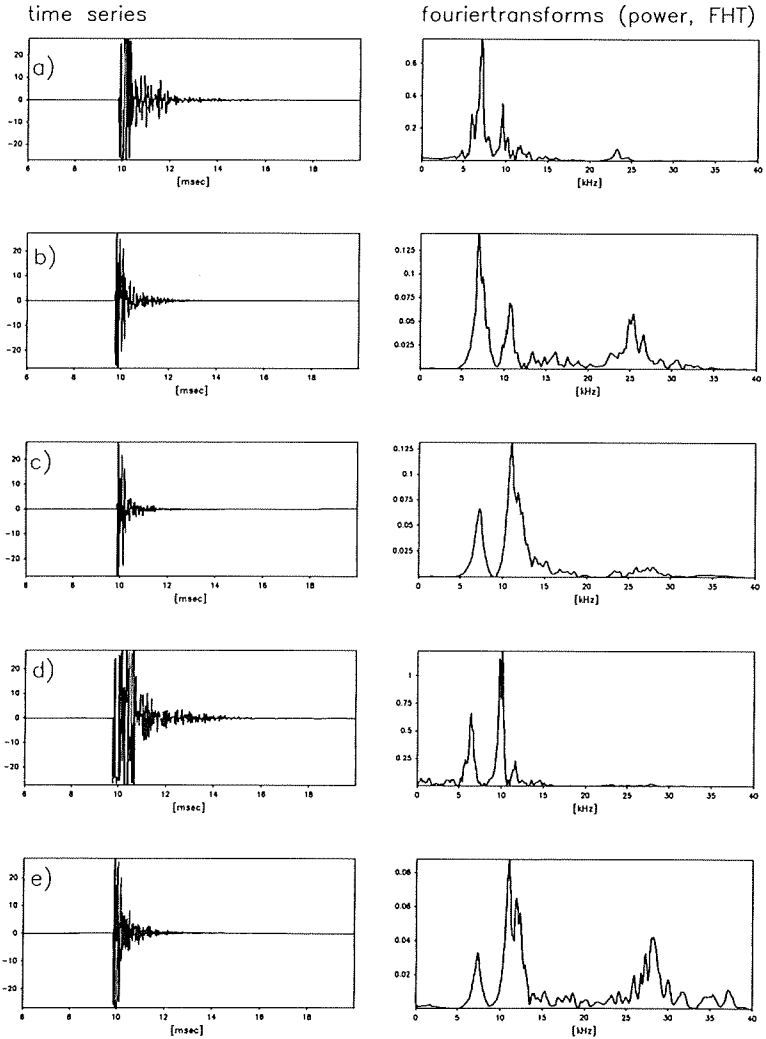
Figure 7: Using the resonance method at a retaining wall (see table 2).

4.3. Test-measurements on a concrete floor

For testing the suitability of the method on structures with unknown dimensions, a blank test has been performed. A concrete floor of unknown thickness has been tested using the resonance method (see fig. 8). To begin with, in an area of ca. 2 m² the P-wave velocity has been determined and systematically the resonances of the background echoes have been measured. While the v_p -measurements delivered average values of 4200 m/s, the resonance measurements showed unexpected pictures. A selection of them is shown in fig. 9. Two differently significant peaks at about 7000 resp. 11000 Hz are easy to perceive. Using the determined v_p , these peaks are corresponding to a reflection horizon of ca. 20 res. 30 cm. We decided to drill a hole at the position represented by fig. 9 e). At this position, the reflections determined by the resonance method should be near 18.9 cm (7200 Hz) resp. 28.3 cm (11100 Hz), the first value representing the main maximum.

Figure 9:
resonance frequencies of a concrete
floor with unknown thickness

EMPA



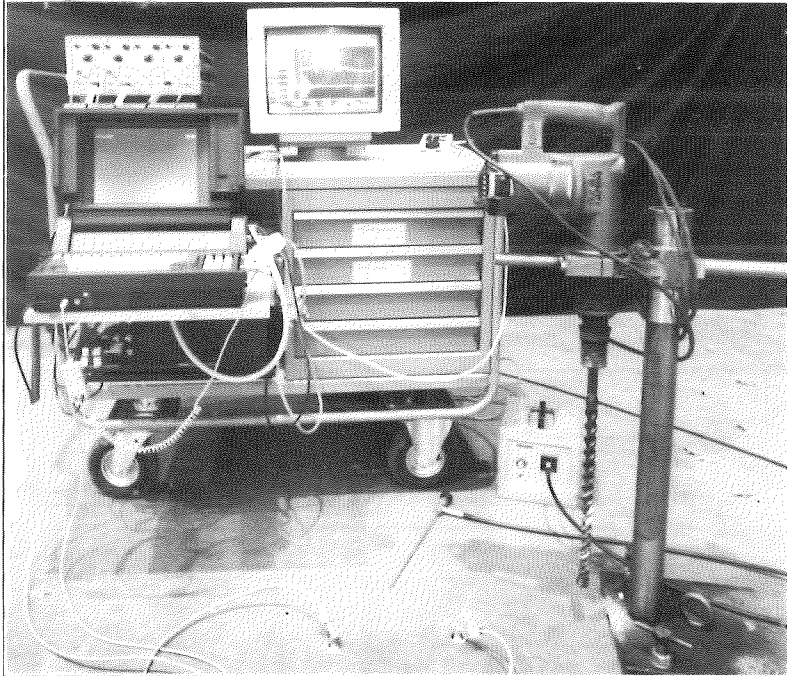


Figure 8: Recording equipment, drilling device and endoscope for the reference measurement.

After that, a hole of 24 mm diameter has been drilled and examined using an endoscope. At approximately 18.5 cm we saw a boundary layer between the concrete slab and a layer of lean concrete. This layer of poor concrete had in fact a thickness of 10 cm, which we did not expect. It was found that the resonance method could very successful predict the depths of reflecting interfaces within measurement accuracy. Obviously, different reflection contrasts between the two layers of concrete at different positions have caused the differences in the amplitudes of the main maxima.

4.4. Estimation of the measurement accuracy and the systematic and statistic errors

Although the measurements provided quite satisfying results, a number of disturbing effects have to be considered. First, it is obviously essential, that the reflectivity at the

back surface (resp. at the reflecting interfaces) is high enough. This means, there has to be a sufficient difference of the material in which the wave is propagating, and the one at which the wave is being reflected. But as we found out, there were no problems, neither at the interface concrete-soil (chapter 4.2), nor at the interface concrete-gravel or concrete-lean concrete (chapter 4.3). Besides, already tests have been successful using this method for the detection of flaws in concrete (with horizontal cracks of some tenths of a millimeter of width) [6, 7] and spalls in sandstone [9]. Cracks in elements of finite thickness are appearing in the frequency spectrum as side maxima with higher frequencies than the main maximum (produced by the background echo).

For a correct interpretation of the frequency spectrum, all the geometric effects are to be known. Especially the lateral boundaries of the objects have to be considered, because they could cause side or even main maxima. By a boundary, for example, additional modes are produced and show up in the frequency spectrum according to the lateral dimensions of the object. As far as these effects are known and do not superpose the frequency area of interest, the measurement is not affected by these problems.

It is important to take into account that the resonance method is not an integral but a point-focal NDT-method. The thickness will be tested at that point, where the transducer is put on. To produce a two-dimensional image of the specimen one has to scan the object with the test equipment. Of course the expense could be significantly higher according to the intended accuracy. Under the assumption that the calculation of v_p is not necessary at every point (see below), a single measurement requires about two minutes. Local inhomogeneities and thickness variations between the test points will not be measured. On the other hand, heterogeneities in the tested part of the object like voids or reinforcement may influence the results.

The error of measurement must be calculated by the law of error propagation. For an experiment, the error function of a determined quantity $f(x_i)$, which is related to some directly measured quantities x_i , is given by:

$$\text{Equation 4: } \Delta f(x_i) = \sum_{i=1}^n \left| \frac{\partial f(x_i)}{\partial x_i} \right| \Delta x_i$$

In our case the main error of the determination of the thickness is calculated according to eq. 3 with:

$$\text{Equation 5: } \Delta d = \frac{\partial d}{\partial s} \Delta s + \frac{\partial d}{\partial f} \Delta f + \frac{\partial d}{\partial t} \Delta t$$

The distance s between the two transducers on the concrete slab surface was fixed to 30 cm. With an accuracy of one millimeter this leads to a relative error of 0.4%.

The accuracy of the frequency analysis is strongly depending on the geometry of the object, which effects the setup for the test. A very important boundary condition is the minimum depth d_{\min} , from which a reflection is expected. The maximum bandwidth of the frequency f_{\max} is related to:

$$\text{Equation 6: } f_{\max} = \frac{v_p}{2 \cdot d}$$

For instance: to measure a thickness of at least 5 cm at a velocity of $v_p = 4000$ m/s the bandwidth must be 40 kHz or more. This corresponds to a sampling rate of 80 kHz respectively a time base of 12.5 μ s or less. Generally we choose at our measure-

ments a time base of $10 \mu\text{s}$ (100 kHz). With a recording interval of 40 ms this is corresponding to approximately 4 kWords of data for one measurement.

The frequency resolution of the fourier transform is fixing the error of the resonance calculations. To achieve an error of less than 1% the resolution f_{min} of the frequency determination should be around 66.7 Hz (according to chap. 4.1: thickness 30 cm and $v_p = 4000 \text{ m/s}$). Actually we obtained a resolution of 25 Hz at a recording interval of 40 ms. This is related to an error of only 0.4 % at frequencies of 6700 Hz. Obviously the error of the frequency measurements is far below the other errors. Especially the P-wave velocity, measured with the travelttime differencies Δt (see eq. 5), is incorrect in a dimension higher. The transient recorder offers a time resolution of $1 \mu\text{s}$ (1 MHz) in maximum, what is related to an error of $2 \mu\text{s}$ (at a thickness of about 30 cm and with an velocity of 4000 m/s) res. 2.7 % at least. In fact the error will be even higher, because of the statistic deviation and the systematic errors like shown in chapter 3.1: the travel path of the signals used for v_p -measurements at the surface is different from that, used for the resonance method. It is difficult to estimate the influence of this error. We found that the effect is in the same order as the error regarding the calculation of v_p with the resonance method (chap. 4.1). At other investigations this effect have to be taken into account.

To draw a conclusion - by using equation 5 - we found an overall error of about 4 %. According to the last paragraph this figure have to be corrected. In our experience it is better to calculate with values of 3-6 % for the error of the thickness measurements. Of cause these values depend on the circumstances and the equipment, so the measuring error have to be calculated for every new experiment.

5. CONCLUSIONS

As shown, at objects of reinforced and plain concrete, a nondestructive determination of the thickness is possible with sufficient precision. The results of the resonance

method are reproduceable with high precision. This seems to enable the detection of delaminations, flaws and voids in concrete.

At present, this method is being improved at the FMPA. To increase the handiness, a shockproof industrial standard laptop equipped with a transient recorder plug-in board has been purchased. This plug-in board enables measurements with 12-Bit resolution and a maximum sampling rate of 5 MHz. Using two parallel inputs, a maximum sampling rate of 2 MHz for each channel is provided. Thereby, above all the determination of the compression wave velocity could have been improved. This has the largest influence to the total error, as shown above. Further researches aim at an improvement of the impact source.

REFERENCES

- [1] Carino, J., Sansalone, M., Hsu, N. Flaw Detection by frequency spectrum analysis of impact-echo waveforms. Int. Adv. in Nondes. Testing, vol. 12, pp. 117-146, 1986.
- [2] Berkhemer, H. Grundlagen der Geophysik. Wissenschaftliche Buchgesellschaft Darmstadt, p. 201, 1990.
- [3] Wüstenberg, H. Möglichkeiten und Konzepte für Ultraschall-Prüfköpfe speziell für das Bauwesen, Int. Symposium Zerstörungsfreie Prüfung im Bauwesen, DGZfP, Berichtsband 21, Teil 2, pp. 488-504, 1991.
- [4] Große, C. Detection of cracks in reinforced concrete - an introduction to the problem with some measurements. Otto-Graf-Journal, Vol.2, FMPA Baden-Württemberg, Stuttgart, pp. 72-90, 1991.
- [5] Sansalone, M., Carino, J. Detecting delaminations in concrete slabs with and without overlays using the impact-echo method. ACI Materials Journal, Vol. 86, Nr. 2, März/April 1989.
- [6] Sansalone, M., Carino, J. Impact-echo: a method for flaw detection in concrete using transient stress waves. National bureau of standards, Gaithersburg, USA, Rep. Nr. 86-3452, September 1986.
- [7] Koo, J.Y., Lee, S., Kim, Y. H., Lew, H.S. Quantitative spectral analysis for flaw detection in concrete. Res. Nondestr. Eval., pp. 171-182, 1991.
- [8] Stearns, S. Digitale Verarbeitung analoger Signale. 2. Auflage, Verlag Oldenbourg, 1984.
- [9] Frick, J., Sawade, G., Große, C. Ultraschallmessungen an Sandsteinfiguren des Ludwigsburger Schlosses. FMPA Baden-Württemberg, unpublished report.