LAY-UP FACTORS FOR WOOD-BASED CROSS-PLY LAMINATES

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SUMMARY

The objective of this paper is to introduce a general approach resulting in lay-up factors for wood-based panels such as Cross Laminated Timber, multi-ply Solid Wood Panels, cross-banded Laminated Veneer Lumber and Plywood. Lay-up factors quantify the effect of the laminate construction on stiffness and strength properties of such wood-based panels and map these on a scale from 0 to 1. After the discussion of some application examples of lay-up factors on the mentioned wood-based panels and a brief survey of the scarce literature on this subject, the derivation of lay-up factors on the basis of a strain energy ratio is shown. Lay-up factors are given for uniaxial tensile or compressive loading as well as for in-plane and out-of-plane bending. For the transverse out-of-plane bending strength, which so far has only been considered by a linear approximation, exact lay-up factors are provided. Lay-up factors that have not yet been specified are derived for in-plane shear as well as for combined out-of-plane bending and in-plane shear. Generally applicable and simplified lay-up factors that only apply under certain assumptions are indicated separately for each type of loading mentioned. The proposed strain energy ratio may also be applied to setting up lay-up factors for hybrid laminates, asymmetric laminate constructions and other combined stress states.

ZUSAMMENFASSUNG

Gegenstand dieses Aufsatzes ist es, einen allgemeinen Ansatz zur Herleitung von Aufbaufaktoren für Holzwerkstoffe wie Brettsperrholz, mehr-lagige Massivholzplatten, Furnierschichtholz mit Querlagen und Sperrholz vorzustellen. Aufbaufaktoren quantifizieren den Einfluss des Lagenaufbaus auf die Steifigkeits- und Festigkeitseigenschaften solcher Holzwerkstoffe und bilden diese auf einer

Skala von 0 bis 1 ab. Nach der Diskussion einiger Anwendungsbeispiele von Aufbaufaktoren auf die genannten Holzwerkstoffe und einem kurzen Überblick über die knappe Literatur zu diesem Thema wird die Herleitung von Aufbaufaktoren auf der Grundlage eines Formänderungs-Energieverhältnisses dargestellt. Aufbaufaktoren werden für einachsige Zug- oder Druckbeanspruchung sowie für Biegung bei Scheiben- und bei Plattenbeanspruchung angegeben. Für die Biegefestigkeit bei Plattenbeanspruchung rechtwinklig zum Faserverlauf der Decklagen, die bislang nur durch eine lineare Annäherung abgebildet wird, werden exakte Aufbaufaktoren hergeleitet. Bislang noch nicht angegebene Aufbaufaktoren werden für Schub bei Plattenbeanspruchung sowie für die Spannungskombination aus Biegung und Schub bei Plattenbeanspruchung hergeleitet. Für die vorstehend aufgeführten Beanspruchungsarten werden jeweils allgemein anwendbare sowie vereinfachte Aufbaufaktoren, die nur unter bestimmten Annahmen gelten, angegeben. Das vorgeschlagene Formänderungs-Energieverhältnis kann ohne Weiteres auch auf die Herleitung von Aufbaufaktoren für Hybrid-Laminate, für unsymmetrische Lagenaufbauten und für weitere Spannungskombinationen angewandt werden.

KEYWORDS: Lay-up factors, wood-based panels, cross-ply laminates, strain energy ratio

1. INTRODUCTION

Wood-based panels with plies aligned in either the longitudinal or transverse direction such as Cross Laminated Timber, multi-ply Solid Wood Panels, cross-banded Laminated Veneer Lumber and Plywood are the preferred materials in timber construction for all those applications where a more uniform load-bearing capacity in both principal material directions or an increased dimensional stability are required. According to the classification schemes of laminate analysis, the wood-based panels mentioned may be regarded as symmetric cross-ply laminates. A laminate is referred to as symmetric when the material, angle and thickness of the constituent plies above and below the mid-plane are the same and a laminate is referred to as a cross-ply laminate when all constituent plies are aligned in either the longitudinal or transverse direction [1].

Wood-based cross-ply laminates are materials with variable stiffness along the laminate thickness or width. The constituent plies are found to have distinctly higher stiffness in the longitudinal than in the transverse direction. The ratio of the moduli of elasticity in the longitudinal and transverse direction for softwood is approximately $E_0/E_{90} \approx 30$ and the associated ratio of the moduli of rigidity is $G_0/G_{90} \approx 14$. The term lay-up factor is used to mean a numerical value that quantifies the effect of the laminate construction on stiffness and strength properties of cross-ply laminates and that maps these on a scale from 0 to 1. Without any claim to completeness, some application examples for lay-up factors are discussed below.

Given two cross-ply laminates with different laminate constructions and nominal thicknesses, lay-up factors allow for a direct comparison with regard to the respective stiffness and strength properties. Lay-up factors thus produce valuable quantitative information for finding optimized laminate constructions for specific load-bearing applications.

The concept of comparing laminate constructions with the help of lay-up factors may be transferred to establish structural design values for cross-banded Laminated Veneer Lumber. Typically, a collective of different laminate constructions and nominal thicknesses is produced here, but for economic reasons usually only a laminate construction from the middle of the thickness range is tested. However, the stiffness and strength properties observed when testing only one laminate construction are not representative of the collective produced and thus may lead to structural design values which are on the unsafe side. The comparison of the tested laminate construction with the other untested laminate constructions using lay-up factors allows quantification of the amount by which the observed stiffness and strength properties must be reduced in order to become representative of the collective. The combination of testing and computational lay-up factor analysis thus leads to increased product safety. The draft of the revised European Standard for Laminated Veneer Lumber [2] provides some guidance on the determination of representative characteristic design values for cross-banded Laminated Veneer Lumber.

A further application of lay-up factors is the determination of construction-independent base values. In some cases, the stiffness and strength properties of a collective of cross-ply laminates with different laminate constructions and nominal thicknesses are available, but the stiffness and strength properties of the ply material, in the context here referred to as base values, are unknown. Base values may be determined from the mentioned data base with the help of lay-up factors. If, conversely, base values are known the stiffness and strength properties of a cross-ply laminate with a specific laminate construction may be found by simply multiplying the base values by their associated lay-up factors. German Technical

Approvals, e. g. [3], for multi-ply Solid Wood Panels make extensive use of these properties. The question of what thickness the transverse plies within multi-ply Solid Wood Panels must have in order to achieve a given proportion of panel stiffness or strength in one of the two principal material directions can also be answered with the help of lay-up factors.

Within the scope of a standard, lay-up factors were specified for the first time in the supplemental sheet to the German plywood standard DIN 68705-5 [4]. These lay-up factors are based upon purely geometric considerations and can be applied to stiffness and strength properties for tensile or compressive loading as well as for in-plane and out-of-plane bending. The major and minor principal material directions are considered separately in the standard. An approximate lay-up factor is given for the transverse out-of-plane bending strength of plywood which accounts for the negligible load-bearing capacity of the outer transverse plies placed in the bending tension zone. The stiffness contribution of the transverse plies to the overall stiffness is not reflected in the equations of the supplemental sheet to DIN 68705-5 [4], which is why their application is limited to wood-based panels with only thin transverse plies such as cross-banded Laminated Veneer Lumber and Plywood.

In their study of the stiffness and strength properties of three- and five-ply Solid Wood Panels, Blaß and Fellmoser [5] suggested closed-form expressions for lay-up factors that reflect the stiffness contribution of the transverse plies to the overall stiffness. Their closed-form expressions are based upon geometrical considerations in conjunction with stiffness-weighting and can be applied to the same stiffness and strength properties as those given in the supplemental sheet to DIN 68705-5 [4].

Contrary to the lay-up factors described above, a generally applicable strain energy ratio is introduced in this paper. The proposed approach leads directly to an exact lay-up factor for the transverse out-of-plane bending strength of cross-ply laminates. Previously not specified lay-up factors for cross-ply laminates under in-plane shear as well as under combined out-of-plane bending and in-plane shear are also readily obtained. Although of primary interest here, the application of the strain energy ratio is not limited to symmetric cross-ply laminates. The strain energy ratio may also be applied to setting up lay-up factors for hybrid laminates made-up with constituent plies of different materials, for asymmetric laminate constructions and for other combined stress states.

2. DEFINITION OF LAY-UP FACTORS

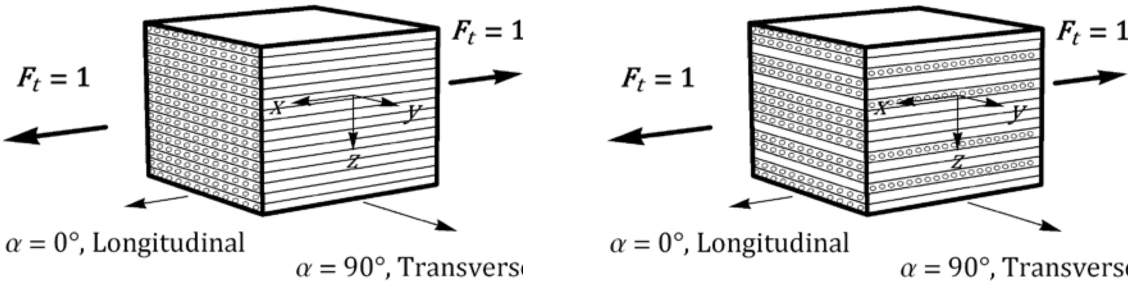
A general approach to obtaining lay-up factors is to compare a unidirectional material as a reference with a multi-directional material. In the sequel, the reference material will be referred to as unidirectional laminate, the constituent plies of which are aligned exclusively in the longitudinal direction as exemplarily seen in Fig. 1a. Most wood-based panels are bidirectional materials. For that reason, the multi-directional material will be constrained to a bidirectional symmetric cross-ply laminate, the constituent plies of which are aligned in either the longitudinal or transverse direction as exemplarily seen in Fig. 1b. Both laminates have the same cross-sectional dimensions and only for reasons of simplification it is assumed that they are made-up of the same ply material. The material law of all plies is transversely isotropic, the Poisson´s ratios being neglected. The load directions are only distinguished in relation to the major and minor principal material directions where the subscript α with the angles $\alpha = 0°$ or $\alpha = 90°$ will be used to indicate these two directions.

Lay-up factors are dimensionless rational or real numbers in the range of $0 < k_\alpha \leq 1$. A lay-up factor of $k_0 = 1$ denotes a unidirectional laminate under any stress acting in the longitudinal direction (see e.g. Fig. 1a) and a lay-up factor of $k_{90} \approx 0$ denotes a unidirectional laminate with negligible transverse stiffness under any stress acting in the transverse direction. Cross-ply laminates with plies aligned in either the longitudinal or transverse direction have therefore lay-up factors in the range of $0 < k_\alpha < 1$.

Lay-up factors may be regarded as an efficiency ratio of the strain energy per unit length $U_{ref,0}$ stored in a longitudinally loaded unidirectional laminate in relation to the strain energy per unit length $U_{ply,\alpha}$ stored in a longitudinally or transversely loaded cross-ply laminate. For the same type and magnitude of load, the associated lay-up factor may be defined as the strain energy ratio

$$k_\alpha = \frac{U_{ref,0}}{U_{ply,\alpha}} \quad . \tag{1a}$$

The consideration of only the unit length does not pose a limitation. It may be shown that the lay-up factor according to Eq. (1a) is independent of the internal force distribution along a bar or beam axis, provided that both the unidirectional and the cross-ply laminate have identical internal force distributions. The solution of the integrals only along the laminate thicknesses then leads to the same lay-up factor as integration over the volumes does.



(a) (b)

Fig. 1 a, b: Longitudinal tensile loading of a unidirectional laminate in (a) and of a cross-ply laminate of same cross-sectional dimensions and of same ply material in (b) each under a unit tensile load "1". Due to the constantly high stiffness of the longitudinal plies, the strain energy $U_{ref,0}$ stored in the unidirectional laminate is lower than the strain energy $U_{ply,\alpha}$ stored in the cross-ply laminate

Fig. 1a shows a unidirectional laminate and Fig. 1b depicts as an example of a cross-ply laminate a laminate construction of cross-banded Laminated Veneer Lumber comprising a total of 16 veneers with four cross-band veneers. Both, the unidirectional and the cross-ply laminate each are shown under a unit tensile force *"1"* acting in the longitudinal direction. The x- and y-axes indicate the longitudinal and transverse direction, while the z-axis runs along the laminate thickness, starts with mid-plane at zero and is positive in downward direction.

As will be seen in the next portion, stiffness always occurs in the denominator of the equations for strain energies. As a consequence, under identical loadings the strain energy per unit length $U_{ref,0}$ stored in a unidirectional laminate with plies of constant high stiffness along the laminate thickness is always lower than the strain energy per unit length $U_{ply,\alpha}$ stored in a cross-ply laminate with plies of high and low stiffness along the laminate thickness. Since $U_{ref,0} < U_{ply,\alpha}$ applies, lay-up factors of cross-ply laminates according to Eq. (1a) are always in the range of $0 < k_\alpha < 1$, as previously pointed out. This conclusion applies to the major and minor principal material directions.

Lay-up factors according to Eq. (1a) may be simplified to the general form

$$k_\alpha = a \cdot \frac{1}{U_{ply,\alpha}} \tag{1b}$$

since the strain energies per unit length $U_{ref,0}$ stored in the unidirectional laminate can be obtained as closed-form expressions from elementary mechanics of materials for bars and beams. The use of more accurate strain energy expressions will not be addressed in this paper.

The strain energy per unit length $U_{ply,\alpha}$ stored in a cross-ply laminate, however, must be calculated separately for each laminate construction and depends on both the ply stress and the ply elasticity distribution along the laminate thickness. The ply stress distribution may be found using classical laminate theory [6], shear analogy method [7] or any other suitable method. The assumption of a constant or linear variation of strain (“transformed section method”) may also apply. As will be shown, the equations for lay-up factors greatly simplify if the latter assumptions along the thickness of a cross-ply laminate hold true. A linear variation of strain under out-of-plane bending is only applicable where the shear deformations are small enough to validate this assumption.

3. LAY-UP FACTORS FOR SOME LOADINGS

3.1 TENSILE OR COMPRESSIVE LOADING, IN-PLANE BENDING

The uniaxial strain energy per unit length of an isotropic, linear elastic and homogeneous bar under tensile or compressive loading may be found from

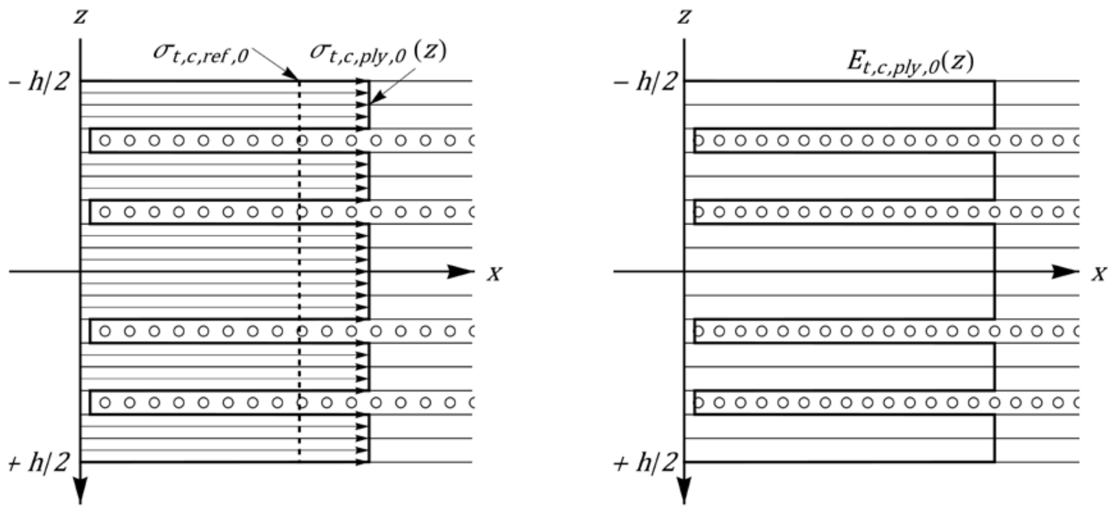
$$U = \frac{1}{2} \cdot \int \frac{\sigma_{t,c}^2}{E_{t,c}} dz \tag{2}$$

where $\sigma_{t,c}$ denotes the tensile or compressive stress and $E_{t,c}$ denotes the tensile or compressive modulus of elasticity. In all what follows, the cross-sectional dimension “width” will be assumed to be "*1*".

According to the strain energy ratio defined in Eq. (1a), the lay-up factor for tensile or compressive loading of a cross-ply laminate takes the form

$$k_{t,c,\alpha} = \frac{U_{ref,0}}{U_{ply,\alpha}} = \frac{\int \frac{{\sigma_{t,c,ref,0}}^2}{E_{t,c,ref,0}} dz}{\int \frac{\sigma_{t,c,ply,\alpha}(z)^2}{E_{t,c,ply,\alpha}(z)} dz} \quad . \tag{3a}$$

In Eq. (3a), $\sigma_{t,c,ref,0}$ and $E_{t,c,ref,0}$ each denote the uniformly distributed normal stress and the modulus of elasticity of the unidirectional laminate. The ply normal stress distribution $\sigma_{t,c,ply,\alpha}(z)$ and the ply elasticity distribution $E_{t,c,ply,\alpha}(z)$ of the cross-ply laminate are typically piecewise constant functions along the laminate thickness.



(a) (b)

Fig. 2 a, b: Longitudinal uniaxial tensile or compressive loading. Qualitative uniformly distributed normal stress $\sigma_{t,c,ref,0}$ (dashed line) of the unidirectional laminate according to Fig. 1 a and ply normal stress distribution $\sigma_{t,c,ply,0}(z)$ (solid polyline) of the cross-ply laminate according to Fig. 1 b due to longitudinal uniaxial tensile or compressive loading in (a) and associated ply elasticity distribution $E_{t,c,ply,0}(z)$ along the laminate thickness in (b)

For the unidirectional and the cross-ply laminate depicted in Figs. 1a, b, Fig. 2a shows the uniformly distributed normal stress $\sigma_{t,c,ref,0}$ (dashed line) and the ply normal stress distribution $\sigma_{t,c,ply,0}(z)$ (solid polyline) along the laminate thickness qualitatively under uniaxial tensile or compressive loading with face plies being aligned in the longitudinal direction. Fig. 2b shows the associated piecewise constant ply elasticity distribution $E_{t,c,ply,0}(z)$. For the graphical representations, a constant strain along the cross-ply laminate thickness and a ratio of the moduli of elasticity $E_0/E_{90} = 30$ for the plies being aligned in either the longitudinal or transverse direction were assumed.

The strain energy per unit length stored in a unidirectional laminate of thickness h under an external unit axial force acting along the direction of the x-axis (see Fig. 1a) results as the solution of the integral in the numerator of Eq. (3a) to $U_{ref,0} = a = 1/\left(2 \cdot E_{t,c,ref,0} \cdot h\right)$. By the use of Eq. (1b), Eq. (3a) may be rewritten to the following general form

$$k_{t,c,\alpha} = \frac{1}{E_{t,c,ref,0} \cdot h} \cdot \frac{1}{\int \frac{\sigma_{t,c,ply,\alpha}(z)^2}{E_{t,c,ply,\alpha}(z)} dz} \quad . \tag{3b}$$

If the assumption of a constant strain along the cross-ply laminate thickness holds true (see Figs. 2a, b), the ply normal stress distribution may be written as

$\sigma_{t,c,ply,\alpha}(z) = n_{E,\alpha}(z) \cdot F/A_{ply,\alpha} = n_{E,\alpha}(z) \cdot 1/\int n_{E,\alpha}(z)dz$. Inserting the latter expression in Eq. (3b) yields, after some rearrangements, the lay-up factor for a cross-ply laminate under uniaxial tensile or compressive loading in its simplified form

$$k_{t,c,\alpha} = \frac{A_{ply,\alpha}}{A_{ref,0}} = \frac{1}{h} \cdot \int n_{E,\alpha}(z)dz \tag{3c}$$

where $A_{ply,\alpha}$ is the stiffness-weighted cross-sectional area of the cross-ply laminate and $A_{ref,0}$ is the un-weighted cross-sectional area of the unidirectional laminate. The integrand $n_{E,\alpha}(z) = E_{t,c,ply,\alpha}(z)/E_{t,c,ref,0}$ in Eq. (3c) is the normalized ply elasticity distribution or valency function along the cross-ply laminate thickness in the range of $0 \leq n_{E,\alpha}(z) \leq 1$ with $E_{t,c,ref,0}$ as normalizing longitudinal modulus of elasticity of the unidirectional laminate. Unless otherwise stated, all integrals run from the top of the laminates to the bottom.

Equations (3a to 3c) yield the same numerical values as the corresponding equations in the supplemental sheet to DIN 68705-5 [4] if the modulus of elasticity of the transverse plies is set to zero. If, on the other hand, the modulus of elasticity of the transverse plies is taken into account, the Eqs. (3a to 3c) provide, as expected, the same numerical values as the equations suggested by Blaß and Fellmoser [5].

When using Eqs. (3a, b), the lay-up factor $k_{t,c,\alpha}$ can be found by simply adjusting the distributions of $\sigma_{t,c,ply,\alpha}(z)$ and $E_{t,c,ply,\alpha}(z)$ to the angles $\alpha = 0°$ or $\alpha = 90°$. If Eq. (3c) is applied, it is sufficient to adjust the valency function $n_{E,\alpha}(z)$ accordingly.

Under the uniaxial loading described, the integration path runs along the axis with variable stiffness, without internal lever arms being effective. The same scenario applies for in-plane bending. Although the stress distributions and equations for determining the lay-up factors for uniaxial loading and in-plane bending are distinctly different from one another, under the assumptions of constant or linear variation of strain the same numerical values are obtained for both loadings. Under the assumptions mentioned, the above defined lay-up factor $k_{t,c,\alpha}$ may also be applied to stiffness and strength properties under in-plane bending (see also supplemental sheet to DIN 68705-5 [4] and [5]).

3.2 OUT-OF-PLANE BENDING

The bending strain energy per unit length of an isotropic, linear elastic and homogeneous beam under pure bending may be found from

$$U = \frac{1}{2} \cdot \int \frac{\sigma_m(z)^2}{E_m} dz \tag{4}$$

where $\sigma_m(z)$ denotes the linear bending stress distribution and E_m denotes the modulus of elasticity in bending.

The strain energy per unit length stored in a unidirectional laminate of thickness h under a unit bending moment about the y-axis may be calculated from $U_{ref,0} = a = 12/\left(2 \cdot E_{m,ref,0} \cdot h^3\right)$. By the use of Eq. (1b), the lay-up factor for a cross-ply laminate under pure out-of-plane bending may be written in the following general form

$$k_{m,\alpha} = \frac{12}{E_{m,ref,0} \cdot h^3} \cdot \frac{1}{\int \frac{\sigma_{m,ply,\alpha}(z)^2}{E_{m,ply,\alpha}(z)} dz} \tag{5a}$$

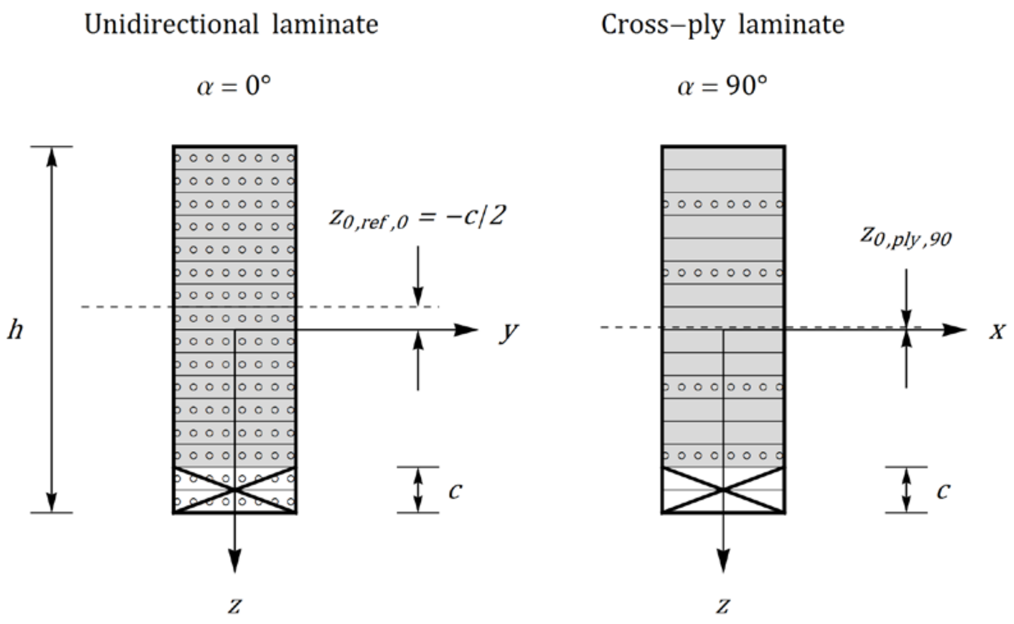
where $E_{m,ref,0}$ is the modulus of elasticity in bending of the unidirectional laminate and $\sigma_{m,ply,\alpha}(z)$ as well as $E_{m,ply,\alpha}(z)$ each denote the ply bending stress and the ply elasticity distribution along the cross-ply laminate thickness.

If the assumption of a linear variation of strain holds true, the lay-up factor for a cross-ply laminate under pure out-of-plane bending takes its simplified form

$$k_{m,\alpha} = \frac{I_{ply,\alpha}}{I_{ref,0}} = \frac{12}{h^3} \cdot \int n_{E,\alpha}(z) \cdot z^2 dz \tag{5b}$$

where $I_{ply,\alpha}$ is the stiffness-weighted second moment of area of the cross-ply laminate and $I_{ref,0}$ is the un-weighted second moment of area of the unidirectional laminate. The expression $n_{E,\alpha}(z)$ in the integrand of Eq. (5b) is again the valency function as previously defined.

If the face plies of a cross-ply laminate are aligned in the longitudinal direction ($\alpha = 0°$), the lay-up factor $k_{m,0}$ may be applied to both the modulus of elasticity in out-of-plane bending and the out-of-plane bending strength.



(a) (b)

Fig. 3 a, b: Transverse out-of-plane bending strength. Shifts in the neutral axes toward the bending compression zones due to omission of outer plies of thickness c in a unidirectional laminate with outer longitudinal tension plies ($\alpha = 0°$) in (a) and in a cross-ply laminate with outer transverse tension plies ($\alpha = 90°$) in (b). The respective residual cross-sections are shown as grey shaded areas

If the face plies of a cross-ply laminate are aligned in the transverse direction ($\alpha = 90°$), the lay-up factor $k_{m,90}$ is only applicable to the modulus of elasticity in out-of-plane bending. The lay-up factor $k_{m,90}$ shall not be applied to the transverse out-of-plane bending strength for the following reason: When the tensile strength perpendicular to the grain of the outer transverse plies placed in the bending tension zone has been exceeded ("first ply failure") they can no longer contribute to the transverse out-of-plane bending strength of the cross-ply laminate, the strength of which is then only controlled by that of the outermost longitudinal tension ply. It seems proper, therefore, to omit the outer transverse tension plies in the cross-section and to carry out all calculations using the residual cross-section of thickness $h - c$ [8, 9].

Figs. 3a, b show the respective cross-sections of the unidirectional laminate ($\alpha = 0°$) and of the cross-ply laminate ($\alpha = 90°$) seen in Figs. 1a, b. In order to obtain identical cross-sectional dimensions, both the outer plies of the unidirectional laminate aligned in the longitudinal direction and the outer plies of the cross-ply laminate aligned in the transverse direction are omitted as illustrated in

Figs. 3a, b. The respective residual cross-sections are shown in Figs. 3a, b as grey shaded areas. Omission of the outer plies in the bending tension zones results in asymmetric residual cross-sections and thus in shifts in the neutral axes from the mid-planes of the unidirectional and of the cross-ply laminate by the amounts $z_{0,ref,0} = -c/2$ and $z_{0,ply,90}$ toward the bending compression zones (see Figs. 3a, b).

An exact lay-up factor for the transverse out-of-plane bending strength may be found by considering the strain energies per unit length stored in the respective residual cross-sections. The application of the strain energy ratio according to Eq. (1a) leads for the bounds of integration from $-h/2$ to $h/2 - c$ to the lay-up factor for the transverse out-of-plane bending strength of a cross-ply laminate in the following general form

$$k_{m,\sigma,90} = \frac{U_{ref,res,0}}{U_{ply,res,90}} = \frac{\int_{-h/2}^{h/2-c} \frac{\sigma_{m,ref,res,0}(z)^2}{E_{m,ref,0}} dz}{\int_{-h/2}^{h/2-c} \frac{\sigma_{m,ply,res,90}(z)^2}{E_{m,ply,res,90}(z)} dz} \quad . \tag{6a}$$

In Eq. (6a), $\sigma_{m,ref,res,0}(z)$ and $E_{m,ref,0}$ each denote the linear bending stress distribution along the residual thickness $h - c$ and the modulus of elasticity of the unidirectional laminate. For a unit bending moment about the y-axis, the strain energy in the numerator of Eq. (6a) may be calculated from $U_{ref,res,0} = a = 12/[2 \cdot E_{m,ref,0} \cdot (h-c)^3]$. The ply bending stress distribution $\sigma_{m,ply,res,90}(z)$ and the ply elasticity distribution $E_{m,ply,res,90}(z)$ of the cross-ply laminate are typically piecewise linear or constant functions along the laminate thickness. In Fig. 4, the linear bending stress distribution (dashed line) and the ply bending stress distribution (solid polyline) are qualitatively plotted along the residual thickness of the cross-sections shown in Figs. 3a, b as grey shaded areas.

If the assumption of a linear variation of strain holds true, the lay-up factor for the transverse out-of-plane bending strength of a cross-ply laminate under pure out-of-plane bending takes its simplified form

$$k_{m,\sigma,90} = \frac{I_{ply,res,90}}{I_{ref,res,0}} = \frac{12}{(h-c)^3} \cdot \int_{-h/2}^{h/2-c} n_{E,90}(z) \cdot \left(z - z_{0,ply,90}\right)^2 dz \tag{6b}$$

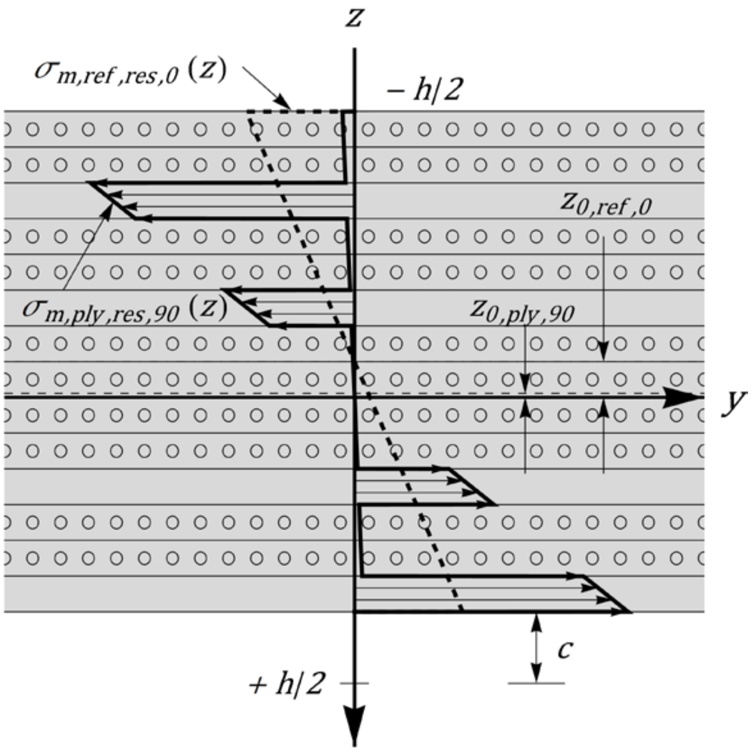


Fig. 4: Transverse out-of-plane bending strength. Qualitative linear bending stress distribution $\sigma_{m,ref,res,0}(z)$ of the unidirectional laminate (dashed line) and ply bending stress distribution $\sigma_{m,ply,res,90}(z)$ of the cross-ply laminate (solid polyline) due to out-of-plane bending of the residual cross-sections shown in Figs. 3a, b as grey shaded areas. Assumptions: Linear variation of strain along the cross-ply laminate thickness, stiffness ratio $E_0/E_{90} = 30$

where $I_{ply,res,90}$ is the stiffness-weighted second moment of area of the cross-ply laminate and $I_{ref,res,0}$ is the un-weighted second moment of area of the unidirectional laminate, both related to the residual thickness $h - c$. The expression $n_{E,90}(z)$ in the integrand of Eq. (6b) denotes again the valency function.

In the computational method according to the supplemental sheet to DIN 68705-5 [4], in which the stiffness contribution of transverse plies is not taken into account (note: in this case $z_{0,ply,90} = 0$ applies), the omission of the outer transverse tension plies is expressed by $k_{m,\sigma,90} = k_{m,90}/\alpha_m$. The correction factor in the denominator is given as $\alpha_m = (h - 2 \cdot c)/h$, which proves to be a good linear approximation of the theoretically exact correction factor $\alpha_{m,exact} = [(h - c)/h]^3$, the derivation of which is not shown in detail here for reasons of space.

3.3 IN-PLANE SHEAR AND OUT-OF-PLANE SHEAR

Neglecting bending stresses, the shear strain energy per unit length of an isotropic, linear elastic and homogeneous beam under shear is

$$U = \frac{1}{2} \cdot \int \frac{\tau_v(z)^2}{G_v} dz \tag{7}$$

where $\tau_v(z)$ denotes the parabolic shear stress distribution and G_v denotes the modulus of rigidity.

Under the same assumptions as before, the shear strain energy per unit length stored in a unidirectional laminate of thickness h may be calculated from $U_{ref,0} = a = \kappa/\left(2 \cdot G_{v,ref,0} \cdot h\right)$, where $\kappa = 6/5$ is the shear correction factor for homogeneous rectangular cross-sections. By the use of Eq. (1b), the lay-up factor for a cross-ply laminate under in-plane shear may be written in the following general form

$$k_{v,\alpha} = \frac{\kappa}{G_{v,ref,0} \cdot h} \cdot \frac{1}{\int \frac{\tau_{v,ply,\alpha}(z)^2}{G_{v,ply,\alpha}(z)} dz} \tag{8a}$$

where $G_{v,ref,0}$ is the modulus of rigidity of the unidirectional laminate and $\tau_{v,ply,\alpha}(z)$ as well as $G_{v,ply,\alpha}(z)$ each denote the ply shear stress and the ply rigidity distribution along the cross-ply laminate thickness.

If the assumption of a linear variation of strain for determining the out-of-plane ply bending stress distribution holds true, the lay-up factor for a cross-ply laminate under in-plane shear takes its simplified form

$$k_{v,\alpha} = \frac{\kappa}{G_{v,ref,0} \cdot h} \cdot \frac{I_{ply,\alpha}^2}{\int \frac{S_{ply,\alpha}(z)^2}{G_{v,ply,\alpha}(z)} dz} \tag{8b}$$

where $S_{ply,\alpha}(z) = \int_z^{h/2} n_{E,\alpha}(\zeta) \cdot \zeta \, d\zeta$ and $I_{ply,\alpha} = \int n_{E,\alpha}(z) \cdot z^2 \, dz$ each are the stiffness-weighted first and second moment of area of the cross-ply laminate under consideration.

By solving the associated area integrals, lay-up factors for out-of-plane shear (= shear stresses that act perpendicular to the panel plane) can be derived analogously. However, these lay-up factors may underestimate the actual out-of-plane shear strengths as they do not reflect the interlocking action resulting from the combination of longitudinal and transverse plies in this loading.

3.4 COMBINED OUT-OF-PLANE BENDING AND IN-PLANE SHEAR

The preceding section dealt with in-plane shear neglecting bending stresses. The proposed strain energy ratio also allows for obtaining a lay-up factor which captures the combination of the stresses due to out-of-plane bending and in-plane shear. The bending moment $M(x)$ and shear force $V(x)$ of a beam subjected to out-of-plane bending and in-plane shear are, of course, functions of the coordinate x, which is assumed to run along the longitudinal axis of the beam. As a consequence, the lay-up factor for combined stresses is no longer a rational or real number, but also becomes a function of x.

The strain energy per unit length at a point x on the longitudinal axis of an isotropic, linear elastic and homogeneous beam under combined out-of-plane bending and in-plane shear is

$$U(x) = \frac{1}{2} \cdot \int \left(\frac{\sigma_m(x,z)^2}{E_m} + \frac{\tau_v(x,z)^2}{G_v} \right) dz \quad . \tag{9}$$

Following the same steps as outlined in the preceding sections, the lay-up factor for a cross-ply laminate of thickness h under combined out-of-plane bending and in-plane shear takes by the use of Eq. (1b) the general form

$$k_{m,v,\alpha}(x) = \left(\frac{12 \cdot M(x)^2}{E_{m,ref,0} \cdot h^3} + \frac{\kappa \cdot V(x)^2}{G_{v,ref,0} \cdot h} \right) \cdot \frac{1}{\int \left(\frac{\sigma_{m,ply,\alpha}(x,z)^2}{E_{m,ply,\alpha}(z)} + \frac{\tau_{v,ply,\alpha}(x,z)^2}{G_{v,ply,\alpha}(z)} \right) dz} \quad \cdot \tag{10}$$

The lay-up factor $k_{m,v,\alpha}(x)$ according to Eq. (10) is consistently in the range of $k_{v,\alpha} \leq k_{m,v,\alpha}(x) \leq k_{m,\alpha}$ for both the major and minor principal material directions. For out-of-plane ply bending stress distributions based upon the assumption of linear variation of strain, no significant simplifications of Eq. (10) were found.

For the cross-banded Laminated Veneer Lumber with face plies being aligned in the longitudinal direction as seen in Fig. 1b, Fig. 5 depicts the graphical representation of the lay-up factor $k_{m,v,0}(x)$ according to Eq. (10) along the x-axis of a beam between its lower threshold $k_{v,0}$ and its upper threshold $k_{m,0}$ using the example of a simply supported beam under a uniform load q. When plotting the curve, a linear variation of strain along the cross-ply laminate thickness was assumed. The input values for performing the numerical evaluation of Eq. (10) are given in the caption of Fig. 5.

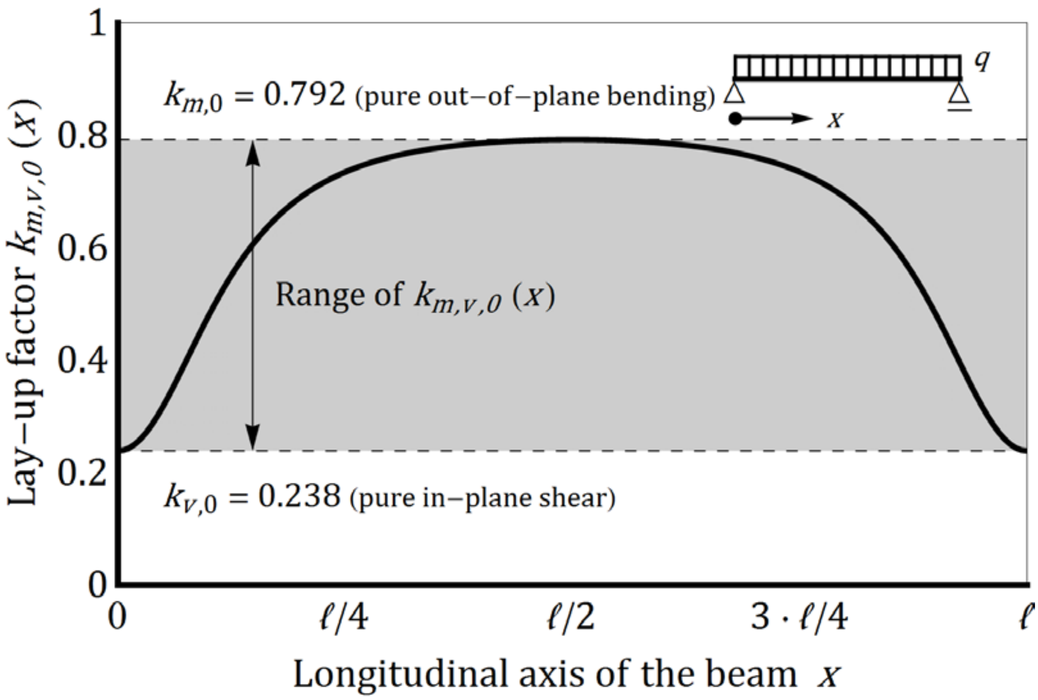


Fig. 5: Combined out-of-plane bending and in-plane shear. For the cross-banded Laminated Veneer Lumber seen in Fig. 1b, under combined out-of-plane bending and in-plane shear the lay-up factor $k_{m,v,0}(x)$ according to Eq. (10) is plotted between its lower threshold $k_{v,0}$ and its upper threshold $k_{m,0}$ as a function of x using the example of a simply supported beam under a uniform load q. Assumptions: Uniform load $q = 1\ N/mm$, span $\ell = 18 \cdot h$, laminate thickness $h = 45\ mm$, stiffness ratios $E_0/E_{90} = 30$ and $G_0/G_{90} = 14$

4. SUMMARY

This paper introduced a general approach resulting in lay-up factors for wood-based panels such as Cross Laminated Timber, multi-ply Solid Wood Panels, cross-banded Laminated Veneer Lumber and Plywood. Lay-up factors quantify the effect of the laminate construction on the stiffness and strength properties of such wood-based panels and map these on a scale from 0 to 1. Lay-up factors thus have a variety of interesting applications. Lay-up factors allow for the comparison of the mechanical properties of two cross-ply laminates with different laminate constructions and nominal thicknesses. For building products where only one laminate construction is tested but a collective of different laminate constructions is produced, lay-up factors help to establish structural design values that are representative of the collective and thus on the safe side. Furthermore, if the mechanical properties of the ply material, also referred to as base values, are known, the mechanical properties of cross-ply laminates with any laminate construction may be determined with the help of lay-up factors.

Contrary to the previously known lay-up factors being based either upon purely geometric considerations or upon geometric considerations in conjunction with stiffness-weighting, the approach for obtaining lay-up factors here was based upon a strain energy ratio. The ratio proposed set the strain energy per unit length stored in a longitudinally loaded unidirectional laminate in relation to the strain energy per unit length stored in a longitudinally or transversely loaded cross-ply laminate. Both laminates had the same cross-sectional dimensions, consisted of the same ply material only for reasons of simplification and were subjected to identical loadings.

For the stiffness and strength properties under tensile or compressive loading as well as under in-plane and out-of-plane bending, generally applicable closed-form expressions of lay-up factors for any ply stress distribution were presented. It also was shown that under the assumption of a constant or linear variation of strain along the cross-ply laminate thickness simplified closed-form expressions for lay-up factors are obtained that, for the loadings mentioned, only require integration along piecewise constant valency functions. In comparison to the lay-up factors given so far, this alleviates the calculation, in particular in the presence of numerous transverse plies in the laminate construction. The simplified lay-up factors may also be written as a product of valency functions with stress distribution ratios or even simply as stress ratios. For reasons of space, these approaches were not pursued further in this paper.

Special attention was given to the transverse out-of-plane bending strength of cross-ply laminates. Under such a loading, the outer transverse plies placed in the bending tension zone do not contribute to the overall strength of the cross-ply laminate. The simplification carried out was to omit the outer transverse tension plies when analysing the laminate construction. For the asymmetric residual cross-section, exact lay-up factors were given, for which there is only a linear approximation so far. Lay-up factors that have not yet been specified were derived for in-plane shear as well as for combined out-of-plane bending and in-plane shear. These lay-up factors supplement the previously known by two further loadings. The proposed strain energy ratio may also be applied to setting up lay-up factors for arbitrarily stacked hybrid laminates, asymmetric laminate constructions and other stress combinations.

In conclusion, it is stated that lay-up factors are merely based upon the distributions of stiffness and stress. Mechanical laws beyond these basic quantities or design-specific particularities, which may also affect the stiffness and strength properties of cross-ply laminates, cannot be represented by these.

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